Chapter 6

ENERGY CONSIDERATION

Introduction:

1.) Newton's Second Law is nice because it provides us with a technique for attacking a certain class of problems. "Focus your attention on the *forces* acting on a body," it says, "and you can deduce something about the body's *acceleration*." As useful as this is, there are other ways to approach motion and physical systems. We are about to develop a new perspective that focuses on the *energy content* of a system.

2.) One of the techniques theoretical physicists use to characterize a physical system is to identify all the parameters (i.e., force or displacement or whatever) that govern a phenomenon of interest, then to multiply those parameters together. The resulting number or vector then acts as a watermark that allows an individual to predict how pronounced the phenomenon in question will be in a particular instance.

a.) Example: What governs the change of a body's velocity? The *force component along the line of motion* certainly matters, and so does the *distance* over which the force is applied. If the product of those two quantities is big, you know the resulting velocity change will be relatively big. If small, the velocity change will be relatively small.

3.) We are about to build a mathematical model that begins with the very product alluded to in *Part 2a*, then looks to see where that definition logically takes us. Hold on to your skirts, ladies. This should be fun.

A.) Work:

1.) The beginning definition: As said above, a change in a body's velocity is governed by the magnitude of the *component of force along the line of the displacement* and the magnitude of the *displacement* itself. The product of these two parameters, $F_{//}$ and d, defines the dot product $\boldsymbol{F}\cdot\boldsymbol{d}$. This quantity is given a special name. It is called *work*.

2.) By definition, the *work* W_F done by a constant force $\boldsymbol{F}$ acting on a body that moves some straight-line distance $\boldsymbol{d}$ (note that $\boldsymbol{d}$ is a vector that defines both the direction and the magnitude of the displacement of the body) is equal to:

$$\begin{aligned} W &= \mathbf{F} \cdot \mathbf{d} \\ &= |\mathbf{F}||\mathbf{d}| \cos \phi, \end{aligned}$$

where ϕ is the angle between *the line of* $\boldsymbol{F}$ and *the line of* $\boldsymbol{d}$.

3.) Example: A box of mass $m = 2\ kg$ moving over a *frictional floor* ($\mu_k = .3$) has a force whose magnitude is $F = 25$ *newtons* applied to it at a 30^o angle, as shown in Figure 6.1 (note that ϕ equals the angle θ in the sketch). The crate is observed to move 16 meters in the horizontal before falling off the table (that is, $\boldsymbol{d} = 16\boldsymbol{i}$ *meters*). An f.b.d. for the forces acting on the block is shown in Figure 6.2.

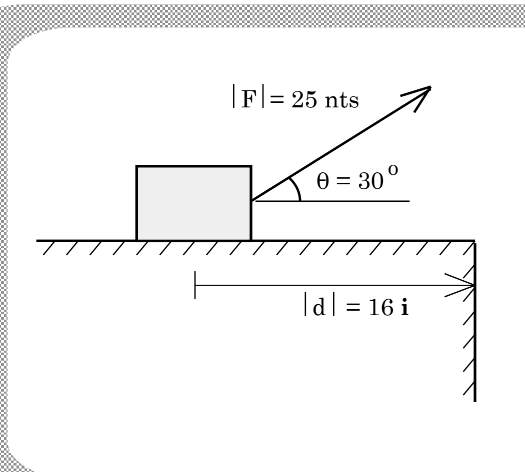


FIGURE 6.1

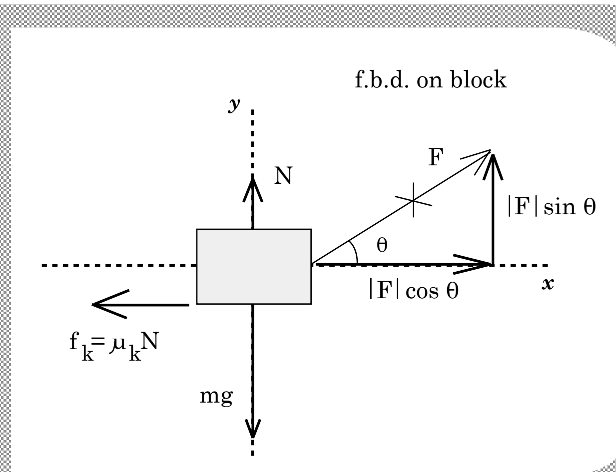


FIGURE 6.2

a.) How much work does $\boldsymbol{F}$ do before the crate takes the plunge?

$$\begin{aligned} W_{\mathbf{F}} &= \mathbf{F} \cdot \mathbf{d} \\ &= |\mathbf{F}| \quad |\mathbf{d}| \quad \cos\theta \\ &= (25 \text{ newtons})\ (16 \text{ meters}) \cos 30^o, \\ &= 346.4 \text{ newton-meters.} \end{aligned}$$

Note 1: A *newton-meter* (or a $kg{\cdot}m^2/s^2$) is the MKS unit for both work and energy. It has been given a special name--the JOULE. We could, therefore, have written *the work done by* F as "346.4 joules."

Note 2: Work and energy units in the CGS system are *dyne-centimeters* (or $gm \cdot cm^2/s^2$). That combination has been given the name ERGES. In the English system, work and energy units are in FOOT-POUNDS.

b.) The above *dot product* was done from a polar notation approach (i.e., you multiplied the *magnitude of one vector* by the *magnitude of the second vector* by the *cosine of the angle between the line-of-the-two-vectors*) because the force information was given in polar notation. If the initial information had been given in *unit vector notation*, you would have used the unit vector approach for the dot product.

For the sake of completeness, let us do the problem from that perspective:

i.) The *unit vector* representation of the force vector presented in our problem above is:

$$\mathbf{F} = (21.65\ \mathbf{i} + 12.5\ \mathbf{j})\ \text{nts}.$$

ii.) *Dot products* executed in *unit vector notation* are defined as:

$$\begin{aligned}\mathbf{F} \cdot \mathbf{d} &= (F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) \cdot (d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \\ &= (F_x\, d_x) + (F_y\, d_y) + (F_z\, d_z).\end{aligned}$$

iii.) As $d_z = 0$, we have:

$$\begin{aligned}W_{\mathbf{F}} &= \mathbf{F} \cdot \mathbf{d} \\ &= (F_x\mathbf{i} + F_y\mathbf{j}) \cdot (d_x\mathbf{i} + d_y\mathbf{j}) \\ &= (21.65\ \mathbf{i} + 12.5\ \mathbf{j}) \cdot (16\ \mathbf{i}) \\ &= (21.65\ \text{newtons})(16\ \text{meters}) + (12.5\ \text{newtons})(0) \\ &= 346.4\ \text{joules}.\end{aligned}$$

This is the same value we determined using the polar approach. As expected, the two approaches yield the same solution.

c.) In our example, how much work does the *normal force* do? The temptation is to assume that we need to determine the magnitude of the normal force before doing this, but a little insight will save us a lot of trouble here. From the definition of *work*:

$$\begin{aligned}W_{\mathbf{N}} &= \mathbf{F} \cdot \mathbf{d} \\ &= |\mathbf{N}||\mathbf{d}|\cos\ \phi.\end{aligned}$$

The trick is to notice that the angle ϕ between $\boldsymbol{N}$ and $\boldsymbol{d}$ is 90^o (see the *free body diagram* shown in Figure 6.2). As $\cos 90^o = 0$, $W_N = 0$.

In fact, *normal forces* are always *perpendicular* to a body's motion. As such, their work contributions will *always* be ZERO. *Normal forces* do no work on a moving body.

d.) How much work does the *frictional force* do on the body as it moves toward the abyss?

i.) To do this part, we need to determine the *normal force N* so that we can determine the *frictional force* using the relationship $f_k = \mu_k N$. Utilizing both the f.b.d. shown in Figure 6.2 and Newton's Second Law:

$$\underline{\Sigma F_y:}$$
$$N + F(\sin\theta) - mg = ma_y.$$

As $a_y = 0$, rearranging yields:

$$\begin{aligned} N &= -F(\sin\theta) + mg \\ &= -(25 \text{ newtons})(\sin 30^o) + (2 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 7.1 \text{ newtons.} \end{aligned}$$

The frictional force will be:

$$\begin{aligned} f_k &= \mu_k N \\ &= (.3)(7.1 \text{ newtons}) \\ &= 2.13 \text{ newtons.} \end{aligned}$$

ii.) Noticing that the angle between the *line of* $\boldsymbol{f_k}$ and the *line of* $\boldsymbol{d}$ is 180^o, the work done by friction will be:

$$\begin{aligned} W_f &= \mathbf{f_k} \cdot \mathbf{d} \\ &= |\mathbf{f}_k||\mathbf{d}| \cos\phi \\ &= (2.13 \text{ newtons})(16 \text{ meters}) \cos 180^o \\ &= -34.1 \text{ joules.} \end{aligned}$$

Note 1: Yes, *work* quantities can be *negative*. In fact, whenever the angle between the *line of* $\boldsymbol{F}$ and the *line of* $\boldsymbol{d}$ is greater than 90^o and less than or equal to 180^o, the cosine of the angle will yield a *negative* number.

Note 2: The *negative sign* is <u>not</u> associated with *direction*. Work is a scalar quantity--IT HAS NO DIRECTION. A negative sign in front of a *work quantity* tells you that the force doing the work is oriented so as to *slow the body down*.

4.) Comments:

a.) Mathematically, the physics concept of *work* is rigidly defined. Hold a 25 pound weight at arm length for fifteen minutes and although there may be sweat pouring off your brow, you will nevertheless be doing no *work*. Why? Because for *work* to occur, a FORCE must be applied to a body *as it moves* over a DISTANCE. If there is no displacement (example: your arm held motionless for fifteen minutes), no *work* is done.

b.) When *work* is done by a single force acting on an object, it *changes the object's motion* (i.e., speeds it up or slows it down). Again, the key is motion. Things become more complicated when many forces act on a body, but in all cases, having some net amount of work being done implies there is motion within the system.

c.) On an intuitive level, forces that do *positive work* are oriented so as to make a body speed up; forces that do *negative work* are oriented so as to make a body slow down. It is as though doing *positive work* puts energy *into* the system while doing *negative work* pulls energy *out of* the system. (We will more fully define the idea of *energy* shortly).

B.) *Work* Due to Variable Forces (A Side Point You Won't Be Tested On):

1.) Let's say you have a ball at the end of a string (Figure 6.3). You apply a horizontal force to the ball to raise it from the vertical to some angle. As you do this, you vary the force so that the ball moves upward on its arc with a constant speed. How do you determine the amount of work *you* have to do to execute this maneuver?

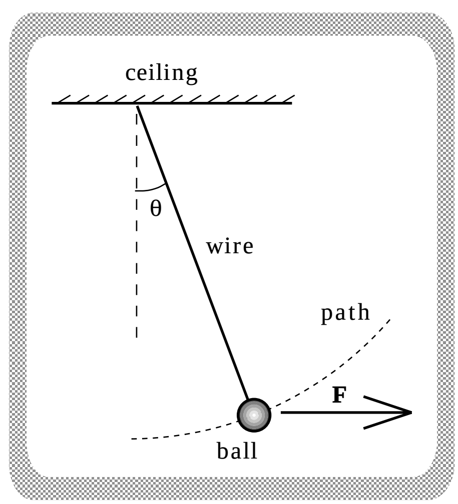


FIGURE 6.3

2.) A more general question is, "When *work* is done by a force-and-displacement combination that in some way varies as a body moves, how do you deal with that?" After all, you can no longer write $W_F = \boldsymbol{F}\cdot\boldsymbol{d}$ and proceed from there. That relationship holds only when all the parameters stay constant throughout the motion.

3.) Answer? Determine the work done over a tiny section--a section in which the displacement (we could write this as a vector $\Delta\mathbf{r}$, but instead will use the notation $d\mathbf{r}$ to denote a very tiny "differential" displacement) is so small that the force is effectively constant over the path. And once done for one little section, do it for the next section, and the next, and the next. And once completed, add them all up to determine the total work done.

4.) In short, we determine the "differential work" (i.e., a very small bit of the overall whole) dW as

$$dW = \mathbf{F}\cdot d\mathbf{r}.$$

5.) Remembering that when you are summing a function that changes continuously (versus one that changes in discrete lumps), the appropriate summation sign doesn't look like Σ, it looks like $\int$, we get the total work--the sum of the differential bits of work--by executing the mathematical operation denoted below (this is called integration, as you probably know). In other words:

$$W = \int dW = \int \mathbf{F} \bullet d\mathbf{r}.$$

6.) There are two ways to evaluate a *dot product*: using a *unit vector* approach and using a *polar* approach. Fortunately for you, you will have to worry about neither!

C.) The Work/Energy Theorem:

1.) We would like to relate the *total, net work* done on an object to its resulting change in *velocity*. This next section is the derivation of just such a relationship.

Note 1: The *work/energy theorem* is a half-way point to where we are really going. Understand it, but also understand that there is a more powerful presentation of the same idea coming up soon.

Note 2: You will not be held responsible for duplicating any of the material you are about to read in this part (i.e., *Part C-1*) except the *bottom line*. It is being provided to give you a chance to see how the Calculus is used in physics derivations, and because it is the easiest way to do the derivation in which we are interested.

Note 3: In short, read this part, not for memorization purposes but for general flow. Do not forget, though, to understand *the bottom line*.

a.) So far, we have been able to calculate the *work* W_F a single force $\boldsymbol{F}$ does on a moving body. It isn't too hard to see that the total, *net work* W_{net} due to all the forces acting on a body will equal the sum of the individual work quantites done by the individual forces.

What might not be so obvious is that there is another way to get that *net work* quantity. How so? We could determine the *net force* $\boldsymbol{F}_{net}$ acting on the body and use *it* in our general *work* definition. Doing so yields:

$$W_{net} = \int \mathbf{F}_{net} \cdot d\mathbf{r}.$$

b.) By Newton's Second Law, the *net force* on an object will numerically equal the vector $m\boldsymbol{a} = m(d\boldsymbol{v}/dt)$. If, for ease of calculation, we assume that the *net force* and the *displacement* $d\boldsymbol{r}$ are both in the $\boldsymbol{i}$ *direction*, we can write the *dot product* associated with the *work* definition as:

$$\begin{aligned} W_{net} &= \int \mathbf{F} \bullet d\mathbf{r} \\ &= \int (m\mathbf{a}) \bullet d\mathbf{r} \\ &= \int \left[m \frac{d(v)}{dt} \mathbf{i} \right] \bullet \left[(dx)\mathbf{i} \right] \\ &= m \int \left(\frac{dv}{dt} \right) dx \qquad \text{(Equation A)} \end{aligned}$$

c.) As the velocity term is *time dependent* (otherwise, we wouldn't be able to determine *dv/dt*), we would like to write the displacement term *dx* in terms of time, also. To do so, note that the *rate at which the position changes with time* (i.e., *dx/dt*) times the *time interval dt* over which the change occurs, yields the *net change in position dx*. Put more succinctly:

$$dx = \left[\left(\frac{dx}{dt}\right)(dt)\right].$$

d.) Substituting this into Equation A and manipulating as only physicists will do (i.e., after canceling out selected *dt* terms), we get:

$$\begin{aligned} W_{net} &= m\int\left(\frac{dv}{dt}\right)\left[\left(\frac{dx}{dt}\right)dt\right] \\ &= m\int dv\left[\frac{dx}{dt}\right]. \end{aligned}$$

e.) Noticing that *dx/dt* is the velocity v of the body, and taking the limits to be from some velocity v_1 to a second velocity v_2, we can rewrite, then integrate this expression as:

$$\begin{aligned} W_{net} &= m\int_{v_1}^{v_2}(v)dv \\ &= m\left(\frac{v^2}{2}\right)_{v_1}^{v_2} \\ &= \left(\frac{1}{2}\right)m(v_2)^2 - \left(\frac{1}{2}\right)m(v_1)^2. \end{aligned}$$

f.) This equation, $W_{net} = (1/2)mv_2^2 - (1/2)mv_1^2$, is called the <u>*Work/Energy Theorem*</u>. IT IS THE *BOTTOM LINE* FOR THIS SECTION.

2.) The quantity $(1/2)mv^2$ has been deemed important enough to be given a special name. It is called the *kinetic energy* of a body of mass m moving with velocity v. Its units are (kg)(m/s)^2, or joules--the same units as *work* (as expected).

a.) OBSERVATION: Something is said to have *energy* if it has the ability to do *work* on another "something."

i.) Example--a car traveling at 30 m/s: A car has energy associated with its motion (i.e., kinetic energy). If this is not obvious, imagine stepping in front of one traveling down the road. Any damage done to you by the car will be due to the fact that the car has energy wrapped up in its motion and, as a consequence, has the ability to do work on you (this gives new meaning to the expression "getting worked").

ii.) Example--a sound wave: If a sound wave didn't carry energy, it wouldn't have *the ability to do work* on the hairs in your ears which, when moved, produce the electrical signals your brain translates into sound.

iii.) In both of the cases cited above, *energy* is associated with *the ability to do work on something else.*

b.) Kinetic Energy--Example #1: What must the *magnitude* of the *velocity* of a 1000 kg car be if it is to have the same *kinetic energy* as a 2 gram bullet traveling at 300 m/s? Keeping the units the same (i.e., converting *grams* to *kilograms* so we can use MKS units), we can write:

<u>Solution:</u>

$$\begin{aligned} KE_b &= (1/2)m_b v_b^{\,2} \\ &= (1/2)\ (.002\text{ kg})\ (300\text{ m/s})^2 \\ &= 90\text{ joules.} \end{aligned}$$

If $KE_b = KE_c$:

$$\begin{aligned} (1/2)\quad m_c \quad v_c^{\,2} &= 90\text{ joules} \\ \Rightarrow\quad (1/2)\ (1000\text{ kg})\ (v_c)^2 &= 90\text{ joules} \\ \Rightarrow\quad v_c &= .42\text{ m/s.} \end{aligned}$$

c.) Kinetic Energy--Example #2: If one triples a body's velocity, how does the body's *kinetic energy* change?

<u>Solution</u>:

$$\begin{aligned} KE_1 &= (1/2)mv_1^{\,2} \\ KE_2 &= (1/2)m(3v_1)^2 \\ &= 9\ [(1/2)m(v_1)^2]. \end{aligned}$$

As would be expected when the *kinetic energy* is proportional to the <u>*square*</u> *of the velocity*, tripling the speed increases the *kinetic energy* by a factor of three-squared, or NINE.

3.) So what does the *Work/Energy Theorem* claim? It maintains that whenever a *net amount of work* is done on a body, the body will either acquire or lose energy. That change will ALWAYS show itself as a change in the *kinetic energy* of the body.

More succinctly: the net work done on a body will always equal the *change of the body's kinetic energy*.

4.) Example: At a given instant, a 2 kg mass moving to the right over a frictional surface has a force $F = 5$ *nts* applied to the left at an angle $\theta = 30^o$ as shown in Figure 6.4. The *average frictional force* acting on the box is $f_k = 1.5$ *nts*. If the block is initially moving with velocity 9 m/s, how fast will it be moving after traveling a distance 4 meters?

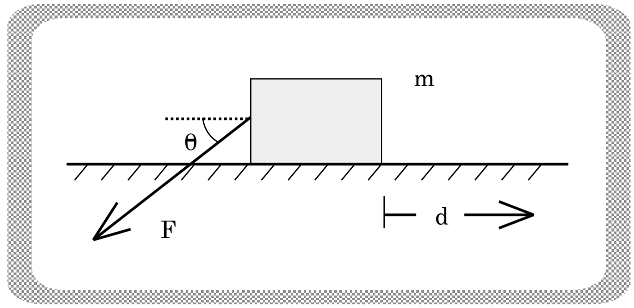


FIGURE 6.4

Note: You could have been given μ_k and been expected to use N.S.L. to determine the normal force N required to use $f_k = \mu_k N$. That twist hasn't been included here for the sake of simplicity, but it is a perfectly legitimate problem for your next test.

a.) Someone well-familiar with the *work/energy theorem* would do the problem as shown below (if the pieces making up the expressions aren't self explanatory, a derivation of each follows in *Part b*):

$$
\begin{aligned}
W_{net} &= \Delta KE \\
W_F + W_f &= \Delta KE \\
(-Fd\cos\theta) + (-f_k d) &= (1/2)mv_2^2 - (1/2)mv_1^2 \\
-(5\text{ nts})(4\text{ m})(.866) - (1.5\text{ nts})(4\text{ m}) &= (1/2)(2\text{ kg})(v_2)^2 - (1/2)(2\text{ kg})(9\text{ m/s})^2 \\
\Rightarrow\quad v_2 &= 7.59\text{ m/s.}
\end{aligned}
$$

b.) The following shows how each quantity used in the above equations was derived:

i.) The *Work/Energy Theorem* states that:

$$W_{net} = \Delta KE.$$

ii.) The left-hand side of the equation is equal to the sum of all the work done by all the forces acting on the block. The f.b.d. shown in Figure 6.5 identifies those forces.

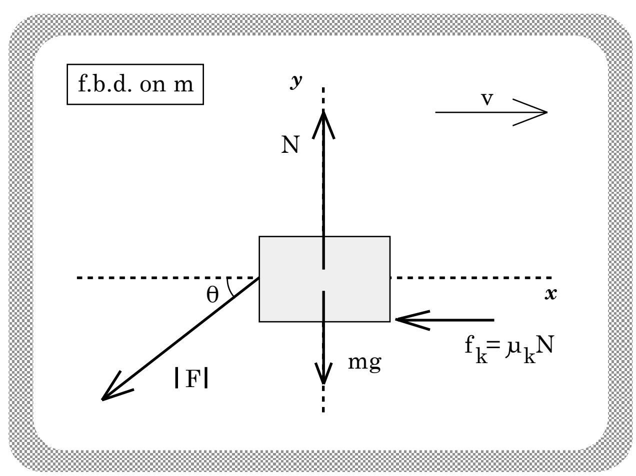


FIGURE 6.5

Note that the work done by the *normal force* will always equal zero (the line of motion and the line of the *normal* are perpendicular to one another). The work due to *gravity* will, in this case, also equal zero for the same reason.

iii.) That leaves $W_{net} = W_F + W_{f_k}$.

iv.) Using the definition of work, we get:

$$\begin{aligned} W_F &= \mathbf{F} \cdot \mathbf{d} \\ &= |\mathbf{F}|\,|\mathbf{d}| \cos\phi \\ &= (F)\,(d) \cos(180^o - \theta) \\ &= -(F)\,(d) \cos\theta. \end{aligned}$$

and

$$\begin{aligned} W_{f_k} &= \mathbf{f}_k \cdot \mathbf{d} \\ &= |\mathbf{F}|\,|\mathbf{d}| \cos\phi \\ &= (f_k)\,(d) \cos 180^o \\ &= -f_k\, d. \end{aligned}$$

Note 1: The angle between *the line of motion* and the force ***F*** is not so obvious--we really did need to write out the *work* derivation for that force.

Note 2: Friction pulls energy out of the system, hence the negative work quantity. That energy is usually dissipated as *heat*.

v.) Putting it all together, we get:

$$\begin{aligned} W_{net} &= W_F + W_{f_k} \\ &= (-Fd\cos\theta) + (-f_k d). \end{aligned}$$

vi.) Returning to the Work/Energy theorem:

$$W_{net} = \Delta KE \qquad \text{(Equation A)}$$

$$\Rightarrow \quad (-Fd\cos\theta) + (-f_k d) = (1/2)mv_2^2 - (1/2)mv_1^2 \qquad \text{(Equation B).}$$

vii.) We know everything except v_2. Solving for that variable, assuming $F = 5$ *newtons*, $f_k = 1.5$ *newtons*, $\theta = 30^o$, $d = 4$ *meters*, $m = 2$ *kg*, and $v_1 = 9$ *m/s*, we get:

$$(-Fd\cos\theta) + (-f_k d) = (1/2)\,mv_2^{\,2} - (1/2)mv_1^{\,2}$$

$$-(5\text{ nts})(4\text{ m})(.866) - (1.5\text{ nts})(4\text{ m}) = (1/2)(2\text{ kg})(v_2)^2 - (1/2)(2\text{ kg})(9\text{ m/s})^2$$

$$\Rightarrow \quad v_2 = 7.59\text{ m/s.}$$

Note 1: Do not memorize the final form of the above equation. The key is to understand *how we got it*. It is the approach that is important here, not the result!

Note 2: Going back for another look at the original formulation of the problem (i.e., the way *you* ought to present a test problem should you be asked to use the *work/energy theorem* to solve a problem):

$$W_{net} = \Delta KE$$

$$W_F \quad + \quad W_f \quad = \quad \Delta KE$$

$$(-Fd\cos\theta) \quad + \quad (-f_k d) \quad = \quad (1/2)mv_2^{\,2} \quad - \quad (1/2)mv_1^{\,2}$$

$$-(5\text{ nts})(4\text{ m})(.866) - (1.5\text{ nts})(4\text{ m}) = (1/2)(2\text{ kg})(v_2)^2 - (1/2)(2\text{ kg})(9\text{ m/s})^2$$

$$\Rightarrow \quad v_2 = 7.59\text{ m/s.}$$

Note 3: For a moment, think about *the approach*. It allows you to relate the total amount of *energy-changing work* W_{net} done on the body to the way the body's energy-of-motion (its *kinetic energy*) changes. Forces come into play in calculating the "work" part of the relationship. That means N.S.L. is still important (you could need it to determine an expression for the magnitude of an unknown force), but the main thrust is wrapped up in the question, "How does the system's ENERGY change?"

Note 4: Although the *Work/Energy Theorem* is important, we will shortly be using it to derive an even more important relationship. We haven't yet gotten to the "bottom line" of this approach.

D.) Conservative Forces:

1.) Background: A body of mass m moves from y_1 (call this Position 1) to y_2 (call this Position 2) with a constant velocity (see Figure 6.6). How much work does *gravity* do on the body as it executes the motion?

Note: There are at least two forces acting on the body in this case, one provided by gravity and one provided

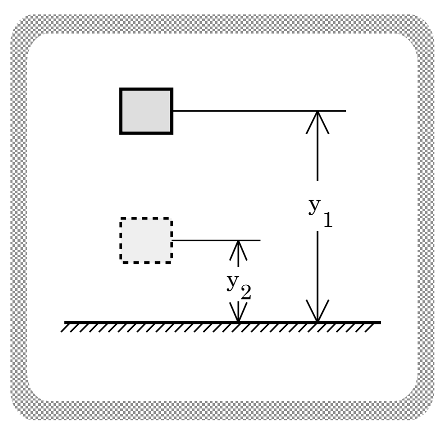


FIGURE 6.6

by an outside agent like yourself. Our only interest in this problem is in the work *gravity* does.

a.) Noting that the angle between the *line of the gravitational force* and the *line of the displacement vector* is 0^o, we can use our definition of *work* to write:

$$
\begin{aligned}
W_{gr} &= \mathbf{F}_g \cdot \mathbf{d} \\
&= |\mathbf{F}|\ |\mathbf{d}| \cos\ 0^\circ \\
&= (mg)\ (y_1 - y_2)(1) \qquad \text{(Equation A),}
\end{aligned}
$$

which could be written:

$$
\begin{aligned}
&= -\ (mg)\ (y_2 - y_1) \\
&= -\ (mg)\ (\Delta y).
\end{aligned}
$$

Note: By definition, Δy is the *final height* y_2 minus the *initial height* y_1. The last two steps of the above derivation were included to make use of this fact (this particular notation will come in handy later).

2.) Let us now replay the situation with a small alteration. Assume now that the block moves from *Point 1* to *Point 2* following the path outlined in Figure 6.7. How much work does *gravity* do on the block in this situation?

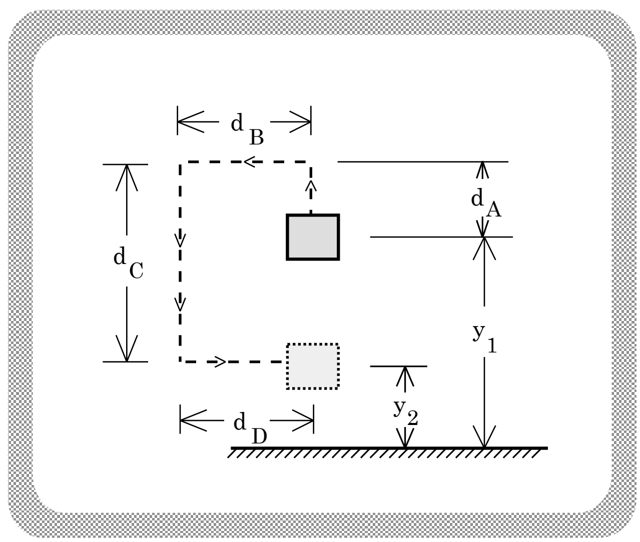


FIGURE 6.7

a.) Noting that the *total work* gravity does will equal the work done by gravity through each section of the displacement, we get:

$$W_{gr} = W_{d_A} + W_{d_B} + W_{d_C} + W_{d_D}.$$

b.) We know that the distance $d_C = d_A + (y_1 - y_2)$. Using that and the definition of *work*, we can write:

$W_{gr} = (mg)(d_A)\cos 180^o + (mg)(d_B)\cos 90^o + (mg)(d_A + y_1 - y_2)\cos 0^o + (mg)(d_D)\cos 90^o.$

c.) Setting *cos 180°= -1*, *cos 90°= 0*, and *cos 0°= 1,* this becomes:

$$\begin{aligned} W_{gr} &= -(mg)\,(d_A) + (mg)\,(d_A + y_1 - y_2) \\ &= -\,(mg)\,(d_A) + (mg)\,(d_A) + (mg)\,(y_1 - y_2) \\ &= +\,(mg)\,(y_1 - y_2) \\ &= -\,(mg)\,(y_2 - y_1). \end{aligned}$$

d.) Notice that this is the same amount of work *gravity* did when the body followed the first path. In fact, no matter what path the body takes in moving from *Point 1* to *Point 2*, the amount of work gravity does on the body will *always be the same*.

Put another way, the *amount of work* gravity does on a body as the body moves from one point to another in the gravitational field is PATH INDEPENDENT. *FORCE FIELDS THAT ACT THIS WAY ARE CALLED CONSERVATIVE FORCE FIELDS*.

e.) A corollary to this *path independence* observation is the fact that the amount of work a *conservative force field* does on a body that moves around a *closed path* in the field will always be ZERO!

Note: "Moving around a closed path" means the body ends up back where it started.

i.) Reasoning? Consider a body that moves upward a vertical distance *d*. The work *gravity* does on the body will be *-mgd* (negative because the angle between the *displacement vector* and the *gravitational force* is 180°). When the body is brought back down to its original position, the work *gravity* does is *+mgd*. The *total work* gravity does on the body as it moves through the round trip is (*-mgd* + *mgd*), or ZERO.

Gravity is a *conservative force field*.

f.) An example of a force field that is not *conservative* is *friction*. Common sense dictates that the further a body moves under the influence of friction, the more work friction will do on the body. As an example, anyone who has ever dragged a fingernail across a chalkboard knows that the further one drags, the more *work* friction does on his fingernails (and the more his listening friends will want to murder him).

From another perspective, frictional forces always *oppose* the direction of relative motion between two bodies. This means that a frictional force will either do all *negative work* or all *positive work* (99% of the time it's negative), depending upon the situation. That, in turn, means that the work due to friction on a body moving around a closed path can never equal zero.

Friction is a *non-conservative force*.

Note: For those of you who are wondering if there are other kinds of non-conservative force fields, all *time-varying* force fields qualify. You will not be asked to deal with time-varying fields until much later; the only *non-conservative* force you will have to worry about for now is friction.

E.) Preamble to the *Gravitational Potential Energy* Function:

Note 1: We are about to consider a concept you have heard about in past science classes but that was most probably never addressed in a truly rigorous way. To eliminate as much intellectual stress as possible, my suggestion is that you forget everything you have ever been told about *potential energy* and start from scratch with the presentation that follows.

Note 2: You will not be held responsible for duplicating any of the material you are about to read except *the bottom line*. BUT, if you don't understand the following material you *won't* understand the *bottom line*, and if you don't understand the "bottom line" you will undoubtedly find yourself totally lost later. Therefore, read the next section, not for memorization purposes but for content. Follow each step as it comes without projecting ahead. When you finally get to the end-result, read back over the material to be sure you know what assumptions were made in proceeding to the endpoint.

1.) Consider a conservative force field--gravity, for instance. A body of mass *m* moves from y_1 (call this Position 1) to y_2 (call this Position 2) with a constant velocity. How much work does the *gravitational force field* do on the body as the body so moves?

a.) This was the question posed at the beginning of the "Conservative Forces" section. The solution was found to be:

$$W_{gr} = -(mg)(y_2 - y_1) \qquad \text{(Equation A).}$$

b.) One of the important conclusions drawn from that section was the observation that as a body moves from *Point 1* to *Point 2* in a gravitational field, the work done by the field is not dependent upon the path taken. Gravity is a *conservative force.*

c.) With that in mind, let's consider a novel idea. If the path counts for nothing--if the endpoints are all that are important when determining the work gravity does--might it not be possible to somehow define a number N_1 that can be attached to *Point 1*, and a number N_2 that can be attached to *Point 2*, and cleverly make them such that the *difference* between them would yield the amount of *work done by gravity* as the body proceeds from *Point 1* to *Point 2*?

d.) This surely is a strange idea, but whether you see the usefulness of it or not, could it be done?

The answer is "yes."

e.) Example: Figure 6.8 shows just such a possible situation. Assuming the numbers have been chosen appropriately, the work done on the body due to gravity as the body goes from *Points 1* to *2* should be:

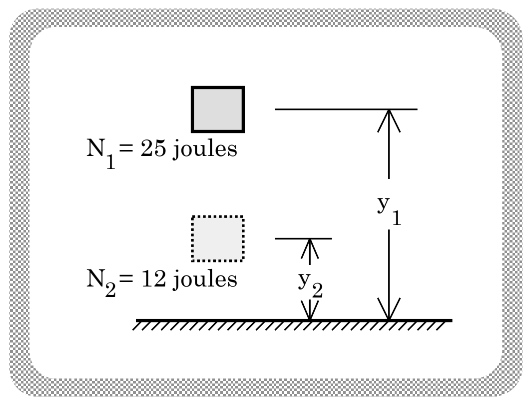


FIGURE 6.8

$$\begin{aligned} W_{gr} &= (N_2 - N_1) \\ &= [(12 \text{ joules}) - (25 \text{ joules})] \\ &= -13 \text{ joules.} \end{aligned}$$

f.) There is only one difficulty with this. We have assigned *zero* to ground level making all numbers above ground level *increase* with elevation. That means that when a body moves from a *higher* (big number) position to a *lower* (small number) position, the difference between the second number and first number ($N_{low} - N_{hi}$) will be negative (just as we found in our example). The problem here is that if we proceed from high to low (i.e., move in the direction of *mg*), the work *gravity* does should be *positive*!

To make our scheme work, we need to modify our original model by re-defining the "numbers expression." We will do so by putting a *negative sign* in front of the relationship. This yields:

$$\begin{aligned} W_{gr} &= -(N_2 - N_1) \qquad \text{(Equation B)} \\ &= -[(12 \text{ joules}) - (25 \text{ joules})] \\ &= +13 \text{ joules.} \end{aligned}$$

g.) With our modification, we now have numbers attached to our initial and final points that, when correctly manipulated, give us the *work done by gravity* as the body moves from *Point 1* to *Point 2*.

Note: Kindly notice that we can do this only because the gravitational force is *conservative* and, hence, the work done due to gravity is *path independent*. If the work done depended upon the path taken, none of this would make any sense at all.

h.) It would be nice to have some handy mathematical function that would allow us to define our *N* numbers. Fortunately, we already have such a function for gravity. Using the definition of work, the *work done by gravity on a body moving from Point 1 to Point 2 in a gravitational field is*:

$$W_{gr} = -(mgy_2 - mgy_1).$$

We determined this expression earlier.

i.) By comparing this equation with our "number expression":

$$W_{gr} = -(N_2 - N_1),$$

we find by inspection that:

$$N_2 = mgy_2 \quad \text{and} \quad N_1 = mgy_1.$$

j.) Written in general (i.e., written as *mgy* where *y* is the <u>*vertical distance*</u> above some arbitrarily chosen *zero-height level*--the ground in our example), this function is important enough to be given a special name. It is called the "*gravitational potential energy*" function, normally characterized as U_g.

k.) <u>Bottom Line</u>: Although we have done this analysis using a gravitational force field, EVERY *conservative force field* has a *potential energy* function associated with it. Furthermore, there is a formal, Calculus-driven approach for deriving *potential energy* functions which will be presented shortly.

Whether you are given a potential energy function or have to derive it, understand that when a body moves through a *conservative force field* the amount of *work done by the field* as the body moves from *Point 1* to *Point 2* will always be:

$$W_{field} = -(U_{pt.2} - U_{pt.1})$$
$$= -\Delta U.$$

This is the bottom line on *potential energy*.

F.) Gravity Close to the Surface of the Earth:

1.) Under normal circumstances, the potential energy function associated with a force field should be zero where the field is zero (you will run into a number of examples of this shortly). The problem with gravity *close to the surface of the earth* is that there *is* no place where the gravitational force is zero. What that means is that *you* can assign the "zero potential energy point" for a given problem.

2.) Put a little differently, gravitational potential energy close to the surface of the earth is not an absolute quantity.

3.) In the off-chance this isn't obvious, consider the table and chalk shown in Figure 6.9.

a.) If we take y to be measured from the table's top (i.e., y_1 in the sketch), we are safe in saying that the amount of *potential energy* the chalk has is equal to mgy_1. If we want to determine the amount of work gravity does on the chalk as it rises to a second point at y_2, we can use the above-derived expression relating *gravitational potential energy* to *the work gravity does*, and get:

$$W_{gr} = -\Delta U_{gr}$$
$$= -(U_2 - U_1)$$
$$= -(mgy_2 - mgy_1).$$

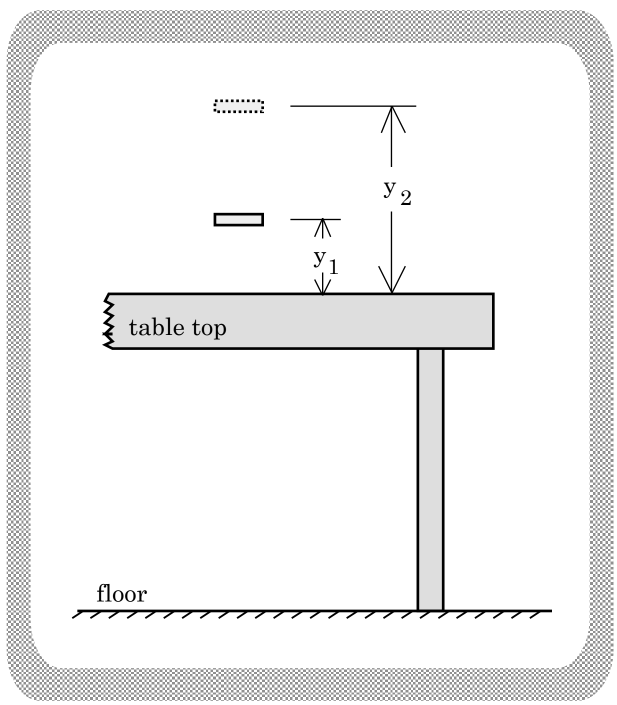


FIGURE 6.9

b.) Could we have used *the floor* as the *zero potential energy level*, making all y measurements from there?

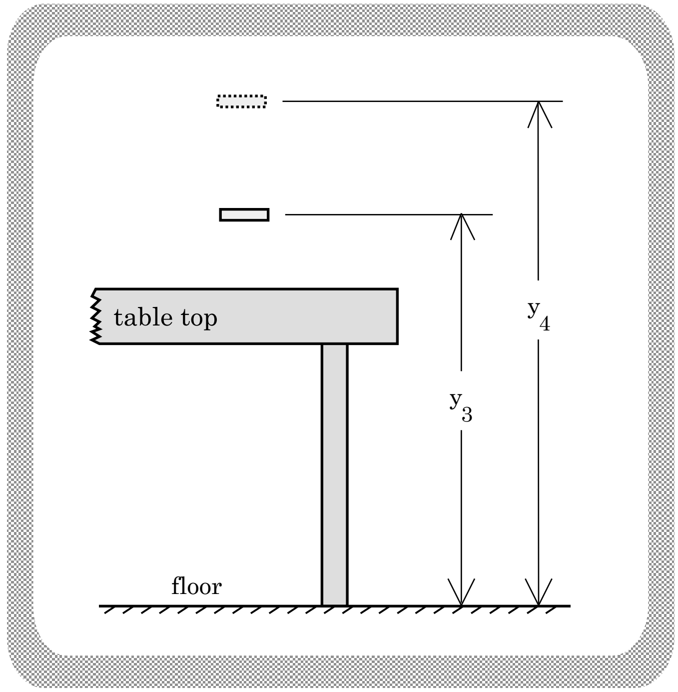


FIGURE 6.10

ABSOLUTELY! The chalk would be assigned an initial *potential energy* value of mgy_3 (see Figure 6.10), etc., and the work calculation would proceed as before:

$$\begin{aligned} W_{gr} &= -\Delta U_{gr} \\ &= -(U_4 - U_3) \\ &= -(mgy_4 - mgy_3). \end{aligned}$$

c.) The amount of work gravity does as the chalk rises to its new position can be determined correctly using *either* approach (notice that $y_2 - y_1$ is numerically equal to $y_4 - y_3$).

Why does this seemingly nonsensical situation exist? Because what is important is not the *amount* of gravitational potential energy an object has when at a particular point. What is important is the *change* of the gravitational potential energy of a body *as it moves from one point to another*. That is what allows us to determine the *amount of work* done on the body as it moves through the gravitational field. Making that *work* determination is the *ONLY USE* you will ever have for *potential energy functions*, ever.

2.) A Work/Energy-Theorem, Potential-Energy Example Problem: A plane oriented at 30^o above the horizontal moves at 300 m/s. It is 1200 meters above the ground when a coke bottle becomes free and sails out of the window *a' la* the movie *The Gods Must Be Crazy* (see Figure 6.11). Neglecting air friction, how fast will the bottle be moving just before it hits the ground?

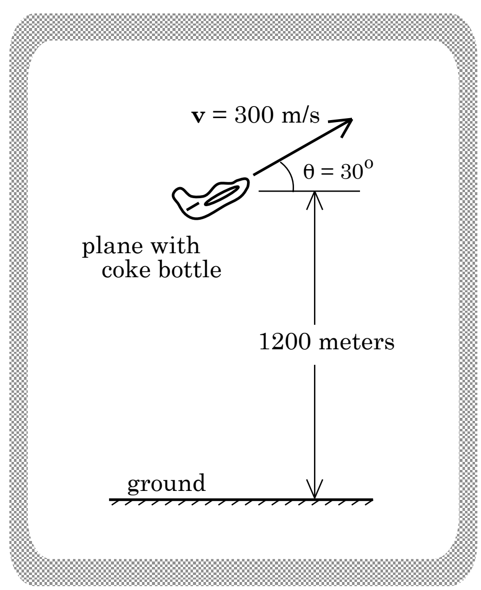


FIGURE 6.11

a.) The work/energy theorem states that the *net work* done on a body must equal the body's change in *kinetic energy* (ΔKE). Mathematically, this is stated as:

$$W_{net} = \Delta KE.$$

b.) In this case, the W_{net} consists solely of the *work done by gravity* W_g. Coupling this with the fact that there is a change in the *kinetic energy* $\Delta KE = (1/2)mv_2^2 - (1/2)mv_1^2$, we can write:

$$W_g = (1/2)mv_2^2 - (1/2)mv_1^2.$$

c.) If we had to calculate the work due to gravity using only the definition, the task would require Calculus (the bottle's *direction of motion* is constantly changing, which means the *angle* between the gravitational force and the displacement is constantly changing--see Figure 6.12), which would be nasty. Fortunately for us, we can easily determine the work gravity does in this situation because:

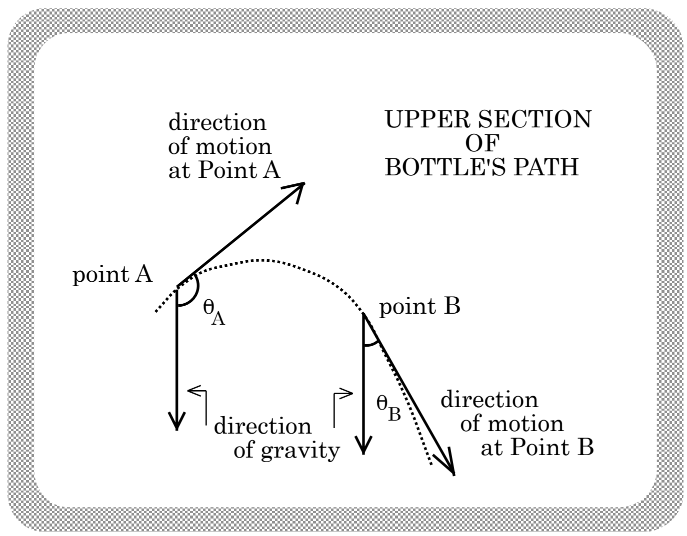


FIGURE 6.12

i.) We know the *potential energy function* for gravity is *mgy*; and

ii.) We know that:

$$W_g = -\Delta U_g = -(U_{2,g} - U_{1,g}).$$

d.) Utilizing these facts, we find:

$$\begin{aligned} W_g &= (1/2)mv_2^2 - (1/2)mv_1^2 \\ -(U_2 - U_1) &= (1/2)mv_2^2 - (1/2)mv_1^2 \\ -(mgh_2 - mgh_1) &= (1/2)mv_2^2 - (1/2)mv_1^2. \end{aligned}$$

e.) Solving for v_2 yields:

$$v_2 = [v_1^2 - 2\,(gh_2 - gh_1)]^{1/2}.$$

f.) Putting in the numbers yields:

$$\begin{aligned} v_2 &= [\,(300\ \text{m/s})^2 - 2\,[(9.8\ \text{m/s}^2)(0\ \text{m}) - (9.8\ \text{m/s}^2)(1200\ \text{m})]\,]^{1/2} \\ &= 336.9\ \text{m/s}. \end{aligned}$$

Note 1: As usual, memorizing this result is a waste of time. What is important is the technique involved. Whenever you need to know *how much work a conservative force does* on a body moving through its force field, that quantity will always equal $-\Delta U$, where U is the *potential energy function* associated with the field.

Note 2: The equation derived using energy considerations and presented above in *Section 2e* should look familiar. Remember $v_2^2 = v_1^2 + 2a(y_2 - y_1)$? It was one of your kinematics equations.

G.) Potential Energy Functions in General:

1.) Although most students associate *potential energy* with *gravitational potential energy*, there are many other *conservative force fields*. For instance, an ideal spring produces a force that is, at least theoretically, conservative. *All conservative forces have potential energy functions associated with them.*

Their use?

If you want to know *how much work* a conservative force field does on a body moving from one point to another within the field, and if you know the field's *potential energy function*, the work done by the field will always equal *minus the change of the potential energy function between the start and end points*, or:

$$W_{cons.force} = -\,\Delta U.$$

H.) FYI, Deriving the *Potential Energy Function* for a Known Force Field (i.e., Something You Will Not Have to Reproduce on a Test):

1.) We created the idea of a *potential energy function* out of the need to easily determine the amount of work gravity does as a body moves from one point to another in a gravitational field. We then concluded that *any* conservative force can have a *potential energy function* associated with it. The only requirement? That:

$$W_{cons.fld.} = -\Delta U \qquad \text{(Equation A)},$$

where the symbol U is used to denote the *potential energy function* associated with the conservative force field with which we happen to be dealing.

2.) Gravity close to the surface of the earth does not have a zero point, but gravity far from the earth does. The force associated with a spring also has a zero point (i.e., its equilibrium point). I could show you the Calculus driven technique by which the potential energy functions for these force fields are derived, but because you won't have to reproduce any of such derivations on a test, I will present only the bottom lines.

3.) The gravitational force between two masses m_1 and m_2 whose *centers of mass* are a distance r units apart is given by the expression

$$\mathbf{F}_g = -G\frac{m_1 m_2}{r^2}\mathbf{r},$$

where G is called the *universal gravitational constant* and $\boldsymbol{r}$ is a unit vector in the *radial* direction (gravitational forces are always directed along a line between the two bodies--i.e., in a *radial* direction).

The question? What is the *potential energy function* for this *force field?*

a.) To begin with, notice that the *force function* is zero when $r = \infty$. This suggests that the *potential energy function* for this force should be zero at infinity.

b.) In fact, the potential energy function for gravity between two object (including the earth and any other object near or far) is

$$U(r) = -G\frac{m_1 m_2}{r}$$

c.) NOTICE: This is the *potential energy function* for gravitational fields anywhere. Does it work? Let's see. According to the theory, we should be able to calculate the amount of work gravity does as a body moves from one point to another in a conservative force field using:

$$W = -\Delta U.$$

Assume your mass is 85 kg. You're in an elevator moving upward from ground level to a position 200 meters above the ground. How much work does *gravity* do as you move?

i.) Using the *gravitational potential energy function* we derived for situations near the surface of the earth (i.e., $U_{mg,near} = mgy$, where we can assume ground level is the *zero potential energy level*), the amount of work done by gravity is found to be:

$$\begin{aligned} W_{grav} &= -[\quad U(y_2 = 200) \quad - U(y_1 = 0)] \\ &= -[\quad mgy_2 \quad - \quad mgy_1 \;] \\ &= -[(85\text{ kg})(9.8\text{ m/s}^2)(200\text{ m}) - \quad 0 \quad] \\ &= -166{,}600 \text{ joules.} \end{aligned}$$

ii.) We would like to do the same problem using the <u>*general*</u> *potential energy function* for gravity (i.e., $-Gm_1m_2/r^2$, where r is the distance between the centers of mass of the interacting objects--in this case, you and the earth). To do so, note that:

--the universal gravitational constant $G = 6.67\text{x}10^{-11}\ m^3/kg{\cdot}s^2$;

--the mass of the earth is $m_e = 5.98\text{x}10^{24}\ kg$;

--the radius of the earth is $r_e = 6.37\text{x}10^6\ m$.

iii.) Remember how *potential energy functions* are used. If you want the amount of work done by a conservative field as a body moves from one point to another in the field, evaluate the *potential energy function* for the field AT THE START POINT and AT THE END POINT, then take *minus* the difference of that amount. The value you end up with will be the work done by the field during the motion. Up until now, all you have worked with has been the simple version of *gravitational potential energy*--a function with an adjustable *zero level*. You are about to use a *potential energy function* with a fixed *zero point* (remember, $U = 0$ at <u>*infinity*</u> for this function). In other words, even though you may be in the habit of treating ground level as the *zero point*, that isn't true of this function! With that in mind:

iv.) If we let r_e be the distance between the earth's center and your center of mass when standing on the earth's surface (this is essentially the radius of the earth), then $r_e + 200$ will be the distance between your *center of mass* when at 200 meters above the earth's surface. We can write:

$$W_{grav} = -[\ U(r_e + 200) \quad - \quad U(r_e)\ ]$$
$$= -\Big[[-Gm_e m_{you}/(r_e + 200)] - [-Gm_e m_{you}/(r_e)]\Big].$$

v.) Pulling out the constants, eliminating the units for the sake of space, and wholly ignoring significant figures, this becomes:

$$\begin{aligned} W_{grav} &= G \quad m_e \quad m_{you}[1/(r_e + 200) \quad - 1/ \quad (r_e) \quad] \\ &= (6.67\text{x}10^{-11})\ (5.98\text{x}10^{24})\ (85)\ [1/(6{,}370{,}200) - 1/(6{,}370{,}000)] \\ &= 5322220652 - 5322387755 \\ &= -167{,}103 \text{ joules.} \end{aligned}$$

vi.) Using the *near Earth potential energy function* in *Part d-i* above, we found that gravity did -166,600 joules of work. If we had not used rounded values for G, r_e, and m_e, these two numbers would have been the same.

vii.) Bottom Line: Our approach for determining *potential energy functions* generates functions that work as expected. LEARN THE APPROACH!

Note: The "near earth" gravitational force on you (i.e., your weight) is $m_{you}g$. So why does the general gravitational force $G\frac{m_{you}m_{earth}}{r^2}$ manage to come out equalling the "near earth" gravitational force $m_{you}g$ when you *are* close to the earth's surface? Remembering that the distance between your center of mass and the earth's center of mass is essentially the radius r_e of the earth, and that this will be true whether you are on the earth's surface or several meters above the earth's surface, the evaluation of the expression $G\frac{m_{earth}}{r_e^2}$ turns out to numerically equal $9.8 m/s^2$. In other words,

$m_{you}\left(G\frac{m_{earth}}{r_e^2}\right) = m_{you}(9.8\,m/s^2) = m_{you}\,g$. Pretty cool, eh?

I.) The Forces Due to and Potential Energy Function of an Ideal Spring:

1.) An ideal spring loses no energy as it oscillates back and forth. The amount of work such a spring does through one full cycle is zero, which is to say that the force it provides is a conservative one. As such, we can derive a *potential energy function* for an ideal spring.

2.) The *position* of a mass attached to an ideal spring is measured from the system's equilibrium position. This is the position at which the force on the mass *due to the spring* is zero. It has been experimentally observed that when a mass is attached to a spring and the spring is elongated or compressed:

a.) The magnitude of the *spring force* exerted on the body is proportional to the spring's displacement from the equilibrium, and

b.) The direction of the force always points toward the equilibrium position.

c.) Assuming the force is in the *x direction,* these observations can be mathematically expressed as:

$$\mathbf{F} = -kx\mathbf{i},$$

where k is a constant that defines the amount of force required to compress the spring *one meter*, and x is the distance the spring is displaced from its equilibrium position (see Figure 6.13).

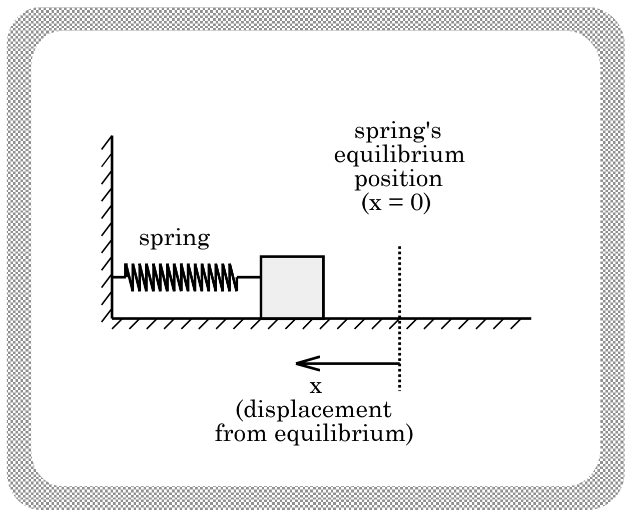


FIGURE 6.13

Note: The displacement term x is really a Δx, but as usual the convention is to assume that the initial position is at $x = 0$. This leaves the displacement term as $\Delta x = x - 0 = x$.

3.) Noting that the force function is ZERO at $x = 0$, convention dictates that the *potential energy function* for a spring must be defined as *ZERO at $x = 0$.*

4.) If we had done the derivation, we would have found that the *potential energy function* for an ideal spring was/is

$$U_{sp} = (1/2)k(x)^2.$$

With that information, we can now use that function in any problem in which an ideal spring does work.

5.) <u>A Problem Involving a *U Function* Other Than Gravity (i.e., that of a Spring):</u> A 2 kilogram block on a horizontal surface is placed <u>without attachment</u> against a spring whose spring constant is $k = 12\ nt/m$. The block is made to compress the spring .5 meters (see Figure 6.14 below). Once done, the block is released and accelerates out away from the spring. If it slides over 2 meters of *frictionless surface* before sliding onto a *frictional* surface, and if it then proceeds to travel an *additional* 13 meters on the *frictional surface* before coming to rest, how large was the *frictional force* that brought it to rest?

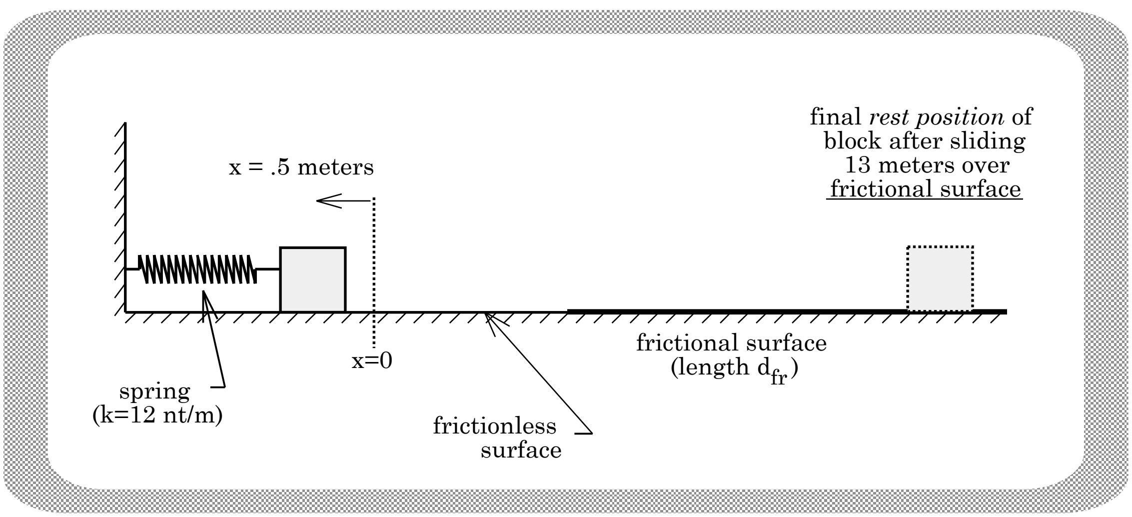


FIGURE 6.14

Note: Do not get too comfortable with using the *work/energy theorem*. It is an OK approach in some cases, but there is a much easier way to deal with the kind of information given in this problem using the concept of *energy conservation*. That alternative approach will be presented shortly. This example is given SOLELY to allow you to see a *potential energy function* other than *gravity* in a problem.

a.) Looking at this problem from a work/energy perspective, we need to determine two different quantities: *the net change of the body's kinetic energy* (i.e., its final *kinetic energy* minus its initial *kinetic energy*), and *the amount of work done by all forces acting on the body* between the beginning and end of its motion. In short, we need to determine:

$$W_{net} = \Delta KE.$$

b.) As the mass does not *rise* or *fall* in this problem, gravity does no work and there is no reason to include the *potential energy function* for gravity in the *work/energy* expression.

c.) Writing this out as you would on a test (should you be asked to use the *work/energy theorem* on a test), we get:

$$
\begin{aligned}
W_{net} &= \Delta KE \\
\Rightarrow \quad W_{sp} + W_{fr} &= KE_2 - KE_1 \\
-\Delta U_{sp} + (-f_k d_{fr}) &= (1/2)\, m\, v_2^{\,2} - (1/2)\, m\, v_1^{\,2} \\
-[0 - (1/2)kx^2] + (-f_k d_{fr}) &= (1/2)\, m\, v_2^{\,2} - (1/2)\, m\, v_1^{\,2} \\
.5(12\text{ nt/m})(.5\text{ m})^2 + (-f_k)(13\text{ m}) &= .5(2\text{ kg})(0)^2 - .5(2\text{ kg})(0)^2 \\
\Rightarrow \quad f_k &= .115\text{ nts.}
\end{aligned}
$$

Note: Once again, THE WORK DONE BY A *CONSERVATIVE FORCE FIELD* ON A BODY MOVING THROUGH THE FIELD WILL ALWAYS EQUAL $-(U_2 - U_1)$, ASSUMING THE *POTENTIAL ENERGY FUNCTION* USED IS THE PROPER FUNCTION FOR THE FORCE FIELD.

J.) MODIFIED CONSERVATION OF ENERGY Theorem: Or, Getting to the Bottom of the Bottom Line:

Note: We are about to put the work/energy theorem into a considerably more useful form. To do so, we will spend some time with the derivation behind "the bottom line." You will not be asked to duplicate this derivation, but if you do not understand it, you will most probably not be able to use the end result to its full extent. Read the following section; think about it; then read it again. It is important that you know what is being done here.

1.) Consider an object with numerous forces acting on it as it moves from *Point A* to *Point B*. The *work/energy theorem* relates the *amount of work done* on the body to the body's *change of kinetic energy*. Writing this out, we get:

$$W_{net} = \Delta KE.$$

The left-hand side of this equation is simply the sum of the work done by all the forces acting on the body. This equation could be written as:

$$W_A + W_B + W_C + W_D + \ldots = \Delta KE,$$

where W_A is the work done by force $\boldsymbol{F}_A$, W_B is the work done by force $\boldsymbol{F}_B$, etc. For the sake of argument:

a.) Assume forces $\boldsymbol{F}_A$ and $\boldsymbol{F}_B$ are *conservative forces* with *known potential energy* functions U_A and U_B. If we define the body's *potential energy* when at *Point 1* due to force field $\boldsymbol{F}_A$ as $U_{A,1}$, and the *potential energy* when at *Point 2* due to force field $\boldsymbol{F}_A$ as $U_{A,2}$, then the *work done* by $\boldsymbol{F}_A$ as the body moves from *Point 1* to *Point 2* in the force field will be:

$$\begin{aligned} W_A &= -\Delta U_A \\ &= -(U_{A,2} - U_{A,1}). \end{aligned}$$

Likewise, the *work done* on the body due to $\boldsymbol{F}_B$ will be:

$$\begin{aligned} W_B &= -\Delta U_B \\ &= -(U_{B,2} - U_{B,1}). \end{aligned}$$

b.) Assume the forces associated with W_C and W_D are either *non-conservative forces* that have no *potential energy function* or *conservative forces* for which we don't know the *potential energy function*. If that be the case, we will have to determine those *work* quantities using:

$$W_C = \mathbf{F}_C \cdot \mathbf{d}$$

and

$$W_D = \mathbf{F}_D \cdot \mathbf{d}.$$

c.) Having made these assumptions, we can write the *work/energy theorem* as:

$$W_A + W_B + W_C + W_D + \ldots = \Delta KE,$$

or

$$[-(U_{A,2} - U_{A,1})] + [-(U_{B,2} - U_{B,1})] + (\mathbf{F}_C \cdot \mathbf{d}) + (\mathbf{F}_D \cdot \mathbf{d}) + \ldots = (1/2)mv_2^2 - (1/2)mv_1^2.$$

d.) Multiplying the *potential energy quantities* by the *-1* outside their parentheses, we get:

$$(-U_{A,2} + U_{A,1}) + (-U_{B,2} + U_{B,1}) + (\mathbf{F}_C \cdot \mathbf{d}) + (\mathbf{F}_D \cdot \mathbf{d}) + \ldots = (1/2)mv_2^{\,2} - (1/2)mv_1^{\,2}.$$

e.) The expression we end up with has:

i.) A number of *potential energy functions* evaluated at t_1 (i.e., when the body is at *Point 1);*

ii.) A number of *potential energy functions* evaluated at t_2 (i.e., when the body is at *Point 2*);

iii.) The *kinetic energy function* evaluated at t_1;

iv.) The *kinetic energy function* evaluated at t_2;

v.) And all the other work done on the body that we haven't been able to keep track of using *potential energy functions*, but that has been done on the body as it moved from *Point 1* to *Point 2.*

f.) If we move all the *time 1* terms to the left-hand side of the equation and all the *time 2* terms to the right-hand side, our equation will look like:

$$(1/2)mv_1^{\,2} + U_{A,1} + U_{B,1} + (\mathbf{F}_C \cdot \mathbf{d}) + (\mathbf{F}_D \cdot \mathbf{d}) + \ldots = (1/2)mv_2^{\,2} + U_{B,2} + U_{A,2}.$$

g.) What we have now is the *kinetic energy* of the body at *Point 1* added to the sum of the *potential energies* attributed to the body while at *Point 1* added to all the extraneous work done on the body (extraneous in the sense that we haven't kept track of it with potential energy functions) between *Points 1* and *2* equaling the *kinetic energy* of the body when at *Point 2* added to the sum of the *potential energies* of the body while at *Point 2.*

Written in shorthand, this is:

$$KE_1 + \Sigma U_1 + \Sigma W_{extraneous} = KE_2 + \Sigma U_2.$$

h.) This is called the *modified conservation of energy* equation. If we identify the sum of the kinetic and potential energies of a body while at a particular point (that is, $KE_1 + \Sigma U_1$) as "the total mechanical energy

E_1" of the body at that point in time, the *modified conservation of energy* equation can be written in an even more compact way:

$$E_1 + \Sigma \, W_{extraneous} = E_2.$$

In this form, the equation states that the *total energy* of the body when at *Point 1* will equal the *total energy* of the body when at *Point 2*, modified only by the "extraneous work" done to the body as it moves *from Points 1 to 2*. In other words, this equation keeps track of the ENERGY the body either *has* or *has-the-potential-of-picking-up* as it moves from one point to another.

Note: The word "conserved" here means "not changing with time." If we have no extraneous bits of work being done as the body moves from *Point 1* to *Point 2*, which is to say we know the *potential energy functions* for all the forces doing work on the body as it moves and there are no non-conservative forces acting on the system, we can write $E_1 = E_2$. This is the true "conservation of energy" equation. That equation is the mathematical way of saying, "The *total energy* of the system will always be the same--the body's *kinetic energy* may change and its *potential energy* may change, but the *sum* of the *kinetic* and *potential energies* will be a constant throughout time."

By adding the possibility of dealing with non-conservative or oddball conservative forces (one for which we haven't a potential energy function), the "modified" *conservation of energy* equation is extremely powerful. It allows for the analysis of situations in which E_1 and E_2 are *not* equal but are related in a deducible way.

2.) Bottom Line: When approaching a problem from the standpoint of energy considerations:

a.) Determine the amount of *kinetic energy* the body has to start with (this may be nothing more than writing down $(1/2)mv_1^2$) and place that information on your sketch next to the body's position at *Point 1*. Do the same for *Point 2*.

b.) Identify any conservative forces for which you know *potential energy functions*. Once identified, determine the *amount of potential* *energy* the body has when at *Point 1* and put that information on your sketch. Do the same for *Point 2*.

Note: If *gravity* is the only force with *potential energy function* in the problem, this last step may amount to nothing more than writing $U_1 = mgh_1$ next to *Position 1* on your sketch with a similar notation at *Position 2*.

c.) Identify any forces that do work on the body as it moves from *Point 1* to *Point 2*, but for which you don't have *potential energy functions*. Determine the *amount of work* they do over the motion and place that information in a convenient spot on your sketch.

d.) Take the information gleaned from *Parts a, b,* and *c,* and after writing out $KE_1 + \Sigma\ U_1 + \Sigma\ W_{extraneous} = KE_2 + \ \Sigma\ U_2$, plug the information in where appropriate. Solve for the unknown(s) in which you are interested.

3.) A Simple Example: Consider a ball of mass .25 kilograms positioned at $y_1 = +4$ *meters* above the ground. It is given an initial upward velocity of 6 m/s at a 60^o angle with the horizontal. The ball freefalls, finally reaching $y_2 = 1$ *meter* above the ground. If friction does 7 joules of work on the ball during the trip, how fast is the ball moving when it gets to $y_2 = 1$ *meter*?

a.) Consider the sketch in Figure 6.15. In it is placed all the information needed to solve this problem. WE WILL ASSUME THE *ZERO POTENTIAL ENERGY* LEVEL is AT THE "FINAL POSITION" (i.e., y_2).

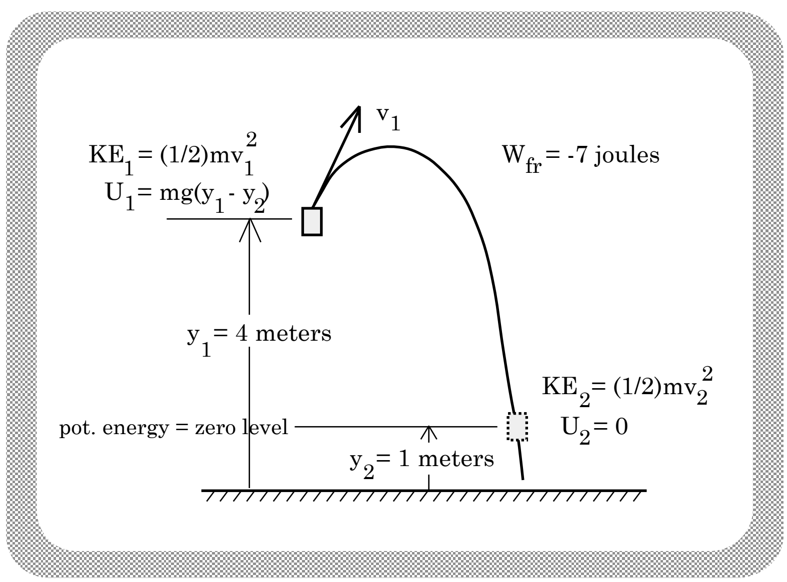


FIGURE 6.15

b.) Remembering that the *work due to friction* is negative and that the *zero potential* *energy level* is at y_2, we can begin with the *modified conservation of energy equation* and write:

$$KE_1 + \Sigma\, U_1 + \Sigma\, W_{extraneous} = KE_2 + \Sigma\, U_2.$$

c.) Spreading out that equation to see what goes where, then solving, we get:

$$\begin{array}{ccccccccc} KE_1 & + & \Sigma\, U_1 & + & \Sigma\, W_{ext} & = & KE_2 & + & \Sigma\, U_2 \\ (1/2)mv_1^{\,2} & + & mg(y_1 - y_2) & + & W_{fr} & = & (1/2)mv_2^{\,2} & + & 0 \\ .5(.25\text{ kg})(6\text{ m/s})^2 & + & (.25\text{ kg})(9.8\text{ m/s}^2)[(4\text{ m}) - (1\text{ m})] & + & (-7\text{ j}) & = & .5(.25\text{ kg})v_2^{\,2} & + & 0 \end{array}$$

$$\Rightarrow \quad v_2 = 6.23\text{ m/s}.$$

Note 1: Important point: Notice the angle had nothing to do with this problem. As far as the concept of energy is concerned, it does not matter whether the body is moving downward or upward or sideways. The amount of energy the body has at a given instant is solely related to the body's mass and velocity, NEVER ITS DIRECTION. As such, do not waste time breaking *velocity vectors* into their component parts. All you need is the velocity's *magnitude*.

Note 2: You could just as easily have taken ground level to be the *zero potential energy level*. If you had, the initial potential energy would have been mgy_1 instead of $mg(y_1 - y_2)$ and the final potential energy would have been mgy_2 instead of zero. Both ways work (if you don't believe me, try it); there is no preferred way to attack the problem.

4.) Example You've Already Seen Done, Done the Easy Way: A 2 kilogram block on a horizontal surface is placed without attachment against a spring whose spring constant is $k = 12\ nt/m$. The block is made to compress the spring .5 meters (see Figure 6.16). Once done, the block is released and accelerates out away from the spring. If it slides over 2 meters of *frictionless surface* before sliding onto a *frictional* surface, and if it then proceeds to travel an *additional* 13 meters on the *frictional surface* before coming to rest, how large is the *frictional force* that brought it to rest?

Note: All the information concerning the *energy state of the system* when the block is at *Point 1* is shown on the sketch. The same is true for *Point 2*. Even the *work done by forces not accommodated by potential energy functions* is written onto the sketch. All the information you need to use the *modified*

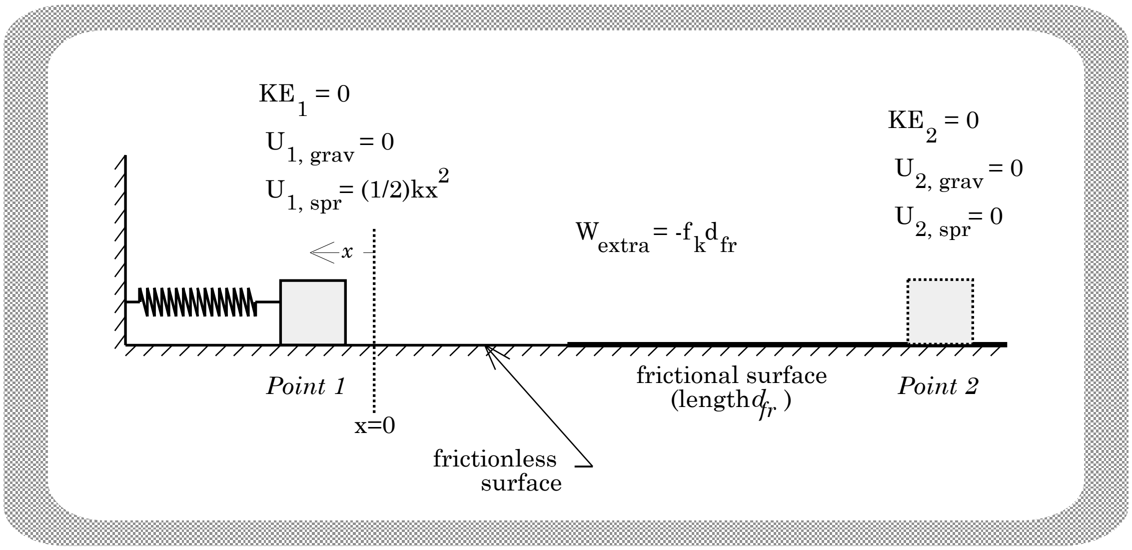


FIGURE 6.16

conservation of energy expression is laid out in its entirety. All that has to be done from there is to put the information into the *c. of e.* equation.

a.) According to the *modified conservation of energy* expression:

$$\begin{array}{ccccccc}
KE_1 & + & \Sigma\, U_1 & + \Sigma\, W_{ext} = & KE_2 & + & \Sigma\, U_2 \\
(1/2)mv_1^{\,2} & + & [U_{1,gr} + U_{1,sp}] & + [W_{f_k}] = & (1/2)mv_2^{\,2} & + & [U_{2,gr} + U_{2,sp}] \\
0 & + & [\;0 + (1/2)kx^2] & + [-f_k d_{fr}] = & 0 & + & [\;0 + 0\;]
\end{array}$$

$$\begin{aligned}
\Rightarrow \quad f_k &= [\;k \quad x^2\;] / [2 \quad d_{fr}\;] \\
&= [(12\text{ nt/m})(.5\text{ m})^2] / [2(13\text{ m})] \\
&= .115\ \text{nts.}
\end{aligned}$$

b.) When this example was done in the *work/energy* section, you were told not to get too attached to the *work/energy* approach. Why? Because another approach was coming that was purported to be easier to use.

You have now seen the other technique--the *modified conservation of energy* approach. What makes it so easy? It is primarily end-point dependent. Indeed, you have to manually determine the amount of work done on the body in-between the end-points if you have forces for which you haven't *potential energy functions*, but that is considerably easier than hassling with *work* calculations for *each* force on an individual basis.

<u>Bottom line:</u> In short, the *modified conservation of energy* approach is easier to execute. Get to know it, understand it, practice it, and you'll learn to love it!

5.) A More Complex Example: A block of mass m is pressed against an unattached spring whose equilibrium position is $d_1 = 3$ *meters* above ground and whose spring constant is $k = 25.6mg/d_1$ (see Figure 6.17). The block is made to compress the spring a distance $d_1/8$ *meters*. The block is additionally forced against the side-wall by your little sister. The force she applies ($\boldsymbol{F_{sis}}$) has a magnitude of $mg/4$ (no, mg does *not* stand for milligrams; it is the *weight of the block--mass* times *gravity*) at an angle of 60^o with the vertical. The wall is *frictional* with a coefficient of friction of $\mu_k = .4$ (see Figure 6.18). Once released by *you* (your sister is still pushing), the block falls. How fast will it be traveling when it reaches *Position 2* a distance $d_1/4$ from the ground?

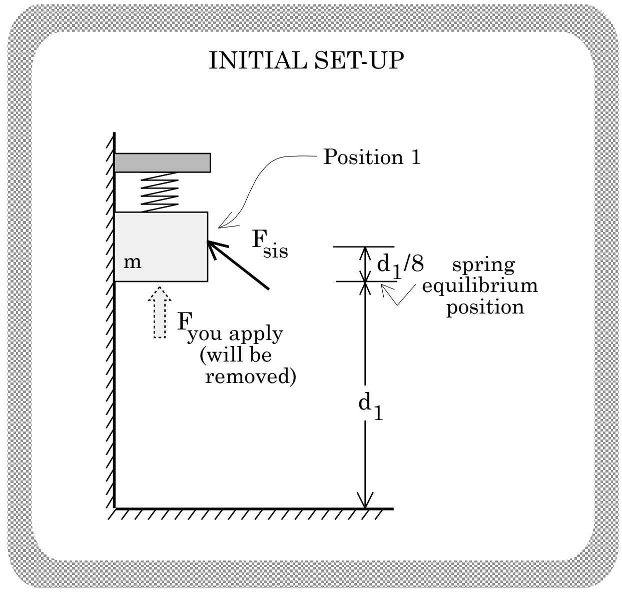


FIGURE 6.17

a.) We need an equation that will allow us to determine the velocity of the block after it has moved to $y = d_1/4$. As the *conservation of energy* approach is related to distances traveled (these are wrapped up in the *work* calculations and *potential energy* functions) and velocities (these are wrapped up in the *kinetic energy* calculations), we will try to use that approach here.

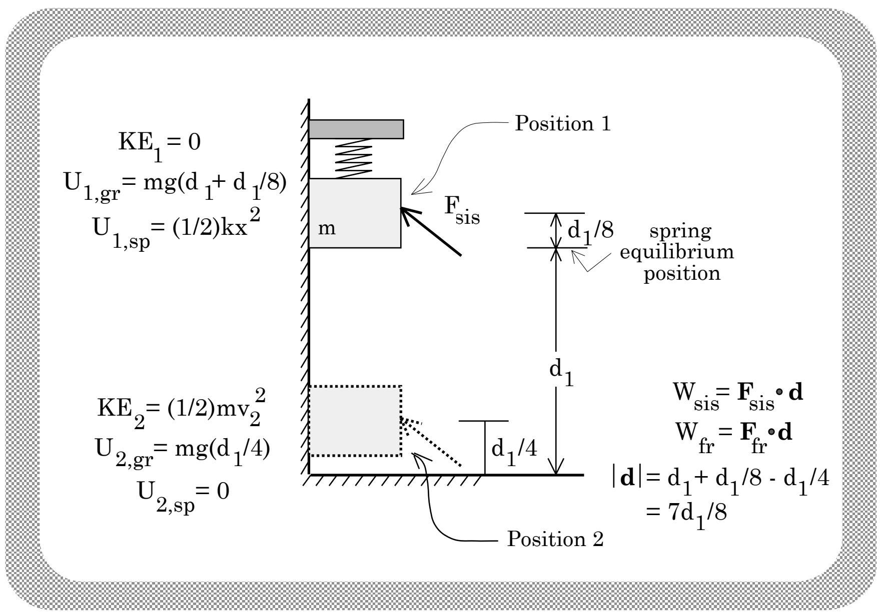


FIGURE 6.18

Note 1: As all our *distance measurements* are relative to the ground, we might as well take the *zero potential energy level* for gravity to be at ground-level.

Note 2: When the block is released, it is accelerated downward by *gravity* and the *spring* but is also retarded in its acceleration by *friction* and *your little sister*. We know *potential energy* functions for *gravity* and the *spring*, but we have no function for *your sister's force* or *friction*.

b.) In its bare bones form, the *modified conservation of energy* equation yields (justification for each part is given below in *Section 5c*):

$$KE_1 + \sum U_1 + \sum W_{ext} = KE_2 + \sum U_2$$

$$0 + [\ U_{1,gr} + U_{1,sp}\ ] + [\ W_{sis} + W_{f_k}\ ] = (1/2)mv_2^{\,2} + [U_{2,gr} + 0]$$

$$0 + [mg(d_1 + d_1/8) + (1/2)kx^2\ ] + [\ \mathbf{F}_{sis} \cdot \mathbf{d}_{sis} + (-f_k)(d_{fr})\ ] = (1/2)mv_2^{\,2} + [mg(d_1/4) + 0]$$

$$0 + [\ mg(9d_1/8) + .5(25.6mg/d_1)(d_1/8)^2] + [(mg/4)(7d_1/8)\cos 120^\circ + (-\mu_k N)(7d_1/8)] = (1/2)mv_2^{\,2} + [.25mgd_1 + 0]$$

$$0 + [(1.125mgd_1) + (.2mgd_1)\ ] + [\ (-.11mgd_1) + (-.074mgd_1)\ ] = (1/2)mv_2^{\,2} + [.25mgd_1]$$

$$\Rightarrow \quad v_2 = [1.78\ gd_1]^{1/2}$$
$$= [1.78\ (9.8\ m/s^2)\ (3\ m)]^{1/2}$$
$$= 7.23\ m/s.$$

c.) If the pieces used in the above expression are obvious, skip this section and continue onward. If they are not obvious, the following should help:

i.) At *Point 1*, as the block is not initially moving:

$$KE_1 = 0.$$

ii.) At *Point 1*, the block has *gravitational potential energy*

$$U_{1,gr} = mg(d_1 + d_1/8) = 1.125mgd_1$$

and *spring potential energy*

$$U_{1,sp} = (1/2)kx^2 = (1/2)(25.6mg/d_1)(d_1/8)^2 = .2mgd_1.$$

iii.) At *Point 2*, the block has gravitational potential energy

$$U_{2,gr} = mg(d_1/4) = .25mgd_1.$$

iv.) At *Point 2*, the block has no *spring potential energy* (as the spring *exerts no force on the block* when the block is at *Point 2*, the spring provides *no potential energy* to the block when at that point):

$$U_{2,sp} = 0.$$

v.) At *Point 2*, the block will have *kinetic energy*

$$KE_2 = (1/2)mv_2^{\,2}.$$

vi.) In between *Points 1* and *2*, "extraneous" work is done by *little sister* in the amount of:

$$\begin{aligned} W_{sis} &= \mathbf{F}_{sis} \cdot \mathbf{d} \\ &= |\mathbf{F}|\,|\mathbf{d}| \cos\phi \\ &= (F_{sis})\,(d) \cos 120^o \\ &= (mg/4)(7d_1/8)(-.5) \\ &= -.11\ mgd_1. \end{aligned}$$

vii.) In between *Points 1* and *2*, "extraneous" work is done by friction in the amount of:

$$\begin{aligned} W_{f_k} &= \mathbf{f}_k \cdot \mathbf{d} \\ &= |\mathbf{F}|\,|\mathbf{d}| \cos\phi \\ &= (f_k)\,(d) \cos 180^o \\ &= f_k(7d_1/8)(-1) \\ &= -.875 f_k d_1. \end{aligned}$$

viii.) To solve this, we need f_k. The easiest way to determine f_k is with Newton's Second Law (the *free body diagram* shown in Figure 6.19 is for the body in mid-flight--it looks a bit different from the fbd for the section of flight during which the spring is still engaged, but the horizontal components are identical in both cases). Doing so yields:

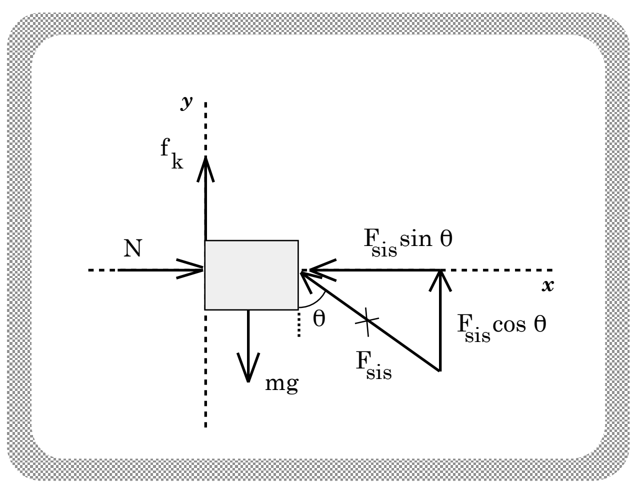


FIGURE 6.19

$\underline{\sum F_x:}$

$$N - F_{sis} \sin\theta = ma_x = 0$$

$$\Rightarrow \quad N = F_{sis}\sin\theta = (mg/4)\sin 60 = .215\ mg.$$

The frictional force is, therefore:

$$f_k = \mu_k N = (.4)(.215\ mg) = .085\ mg.$$

With that, we can determine the work friction does:

$$W_{f_k} = -.875 f_k d_1 = -.875\ (.085\ mg)\ d_1 = -.074\ mgd_1.$$

d.) As we did in the beginning, putting it all together yields:

$$KE_1 + \sum U_1 + \sum W_{ext} = KE_2 + \sum U_2$$

$$0 + [\ U_{1,gr} + U_{1,sp}\ ] + [\ W_{sis} + W_{f_k}\ ] = (1/2)mv_2^2 + [U_{2,gr} + 0]$$

$$0 + [mg(d_1 + d_1/8) + (1/2)kx^2\ ] + [\ \mathbf{F}_{sis}\cdot\mathbf{d}_{sis} + (-f_k)(d_{fr})\ ] = (1/2)mv_2^2 + [mg(d_1/4) + 0]$$

$$0 + [\ mg(9d_1/8) + .5(25.6mg/d_1)(d_1/8)^2] + [(mg/4)(7d_1/8)\cos 120^o + (-\mu_k N)(7d_1/8)] = (1/2)mv_2^2 + [.25mgd_1 + 0]$$

$$0 + [(1.125mgd_1) + (.2mgd_1)\ ] + [\ (-.11mgd_1) + (-.074mgd_1)\ ] = (1/2)mv_2^2 + [.25mgd_1]$$

$$\Rightarrow \quad v_2 = [1.78\ gd_1]^{1/2} = [1.78\ (9.8\ m/s^2)\ (3\ m)]^{1/2} = 7.23\ m/s.$$

K. One More Twist--Energy Considerations with Multiple-Body Systems:

1.) The idea behind the *modified conservation of energy* equation is that it is possible to keep track of not only the amount of energy in a system, but also how the energy is distributed throughout the system.

2.) Up until now, all we have examined have been single-body systems. It is possible to extend the *energy considerations* approach to take into account the energy of a whole group of objects.

3.) Executing this expanded version of the *modified conservation of energy* approach:

a.) Calculate the *total kinetic energy* (i.e., the kinetic energy of each body in the system added together) at time t_1.

b.) To that, add the *total potential energy* (i.e., all potential energy of all sorts acting on each body in the system, all added together) at time t_1.

c.) To that, add the *total extra work* done on all the bodies in the system between times t_1 and t_2.

d.) Put the above sum equal to the *total kinetic energy* plus the *total potential energy* in the system at time t_2 .

4.) The modified *modified conservation of energy equation* thus becomes:

$$\sum KE_{1,tot} + \sum U_{1,\,tot} + \sum W_{extra,tot} = \sum KE_{2,tot} + \sum U_{2,tot}.$$

5.) Example: An Atwood Machine is simply a string threaded over a pulley with a mass attached to each end (see Figure 6.20). Assuming the pulley is ideal (i.e., massless and frictionless) and that $m_1<m_2$, how fast will m_1 be moving if the system begins from rest and freefalls a distance h meters?

a.) The system in its initial state is shown in Figure 6.20. Notice that each body is assigned a *zero gravitational-potential-energy level* of its own.

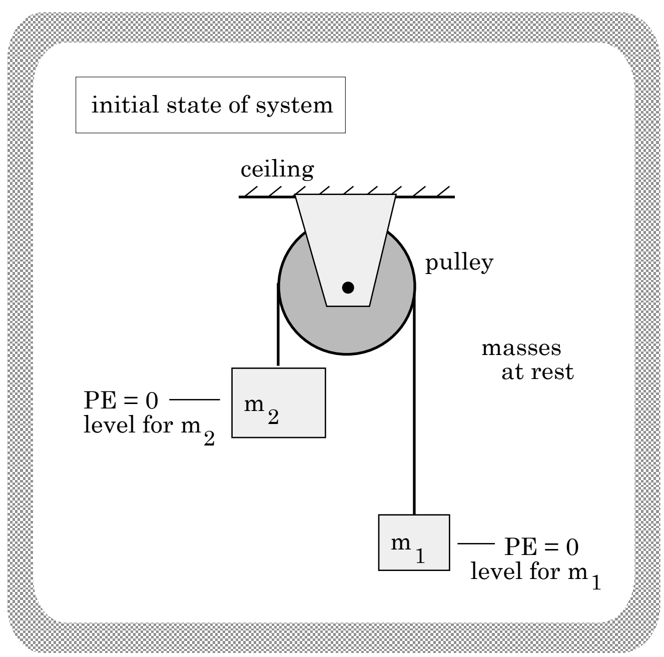


FIGURE 6.20

Note: We could have assigned a common level, but it is easier the other way (remember, *where* the zero is for a given body doesn't matter--it is *changes* in *potential energy* that count).

b.) Figure 6.21 shows the system after the freefall. Notice that mass m_2 has moved *below* its zero-potential-energy-level, making the *potential energy* at that point *negative.*

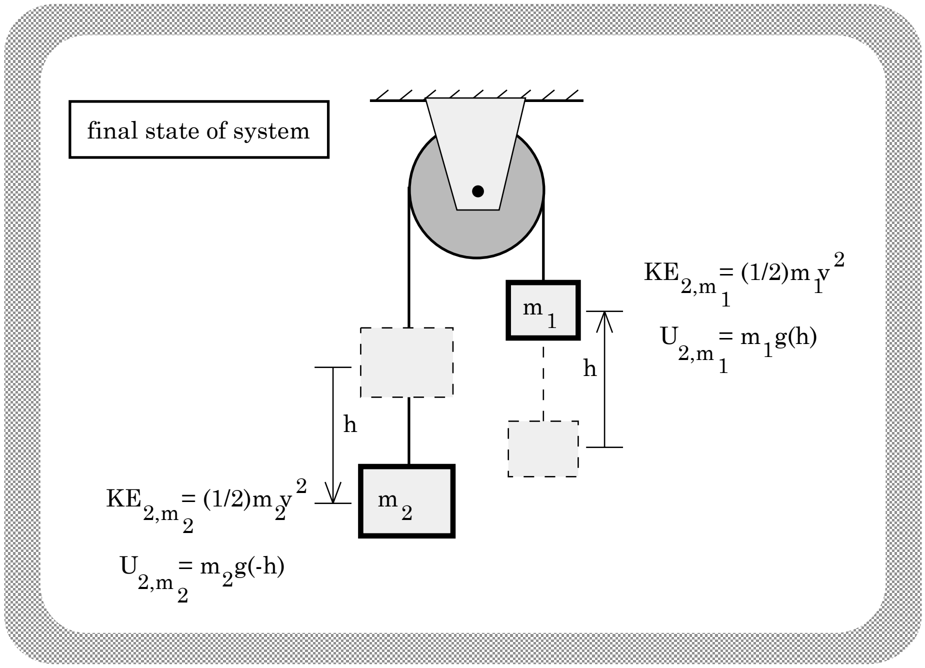


FIGURE 6.21

Note: The amount of *work* tension does on m_1 is $+T(h)$, whereas the amount of *work* tension does on m_2 is $-T(h)$. As such, the two *work quantities* associated with the *tension in the line* add to zero.

c.) Putting everything together and executing the *modified conservation of energy* approach, we get:

$$\sum KE_{1,tot} \quad + \quad \sum U_{1,\,tot} \quad + \quad \sum W_{extra,tot} \quad = \quad \sum KE_{2,tot} \quad + \quad \sum U_{2,tot}.$$

$$[KE_{1,m_1} + KE_{1,m_2}] + [U_{1,m_1} + U_{1,m_2}] + [T(h) + T(-h)] = [KE_{2,m_1} + KE_{2,m_2}] + [U_{2,m_1} + U_{2,m_2}]$$

$$[\,0 + 0\,] \quad + \quad [\,0 + 0\,] \quad + \quad [\,0\,] \quad = [.5m_1v^2 + .5m_2v^2] + [m_1gh + m_2g(-h)]$$

$$\Rightarrow \quad v = [[-m_1gh + m_2gh] \,/\, [.5(m_1 + m_2)]]^{1/2}.$$

L.) Power:

1.) There are instances when knowing how much work is done by a force is not enough. As an example, it may seem impressive to know that a particular motor can do 120,000 joules of work, but not if it takes ten years for it to do so. The amount of *work* per unit time being done is often more important than *how much* work can be done.

2.) The physics-related quantity that measures "work per unit time" is called *power*. It is defined as:

$$P = W/t,$$

where t is the time interval over which the work W is done.

3.) The units for power in the MKS system are $kg \cdot m^2/s^3$. This is the same as a *joules/second,* which in turn is given the special name *watts*. Although the watt is a unit most people associate with electrical devices (the light bulb you are using to read this passage is probably between 60 watts and 150 watts), the quantity is also used in mechanical systems. Automobiles are rated by their *horsepower*. One horsepower is supposedly the amount of *work* a "standard" horse can do *per unit time*. As *formally* defined, one horsepower equals 746 watts.

4.) A special relationship is often derived in physics books that relates the amount of power provided by a force $\mathbf{F}$ as it is applied to a body that moves a distance $\mathbf{d}$ with constant velocity $\mathbf{v}$. Simply presented:

$$\begin{aligned} P_F &= W/t \\ &= (\mathbf{F}\cdot\mathbf{d})/t \\ &= \mathbf{F}\cdot(\mathbf{d}/t) \\ &= \mathbf{F}\cdot\mathbf{v}. \end{aligned}$$

This manipulation has been included for the sake of completeness.

QUESTIONS

6.1) A net force accelerates a body. If you multiply that force by the distance over which it is applied, what will that quantity tell you?

6.2) A net force F stops a car in distance d. In terms of F, how much force must be applied to stop the car in the same distance if its velocity is tripled?

6.3) An object of mass m moving with speed v comes to rest over a given distance d due to the effects of friction. What do you know about the average frictional force involved (i.e., how large must it have been)?

6.4) Two masses, m and $2m$, both freefall from rest. Ignoring friction, which has the greater speed after falling a given distance? Which has more work done to it by gravity over that distance? Is there something to explain here? If so, do so.

6.5) A car slows from 40 m/s to 20 m/s, then from 20 m/s to 0 m/s. In which instance (if any) was more energy pulled out of the system? Reversing the question, going from zero to 20 m/s requires more, the same, or less energy than is required to go from 20 m/s to 40 m/s? Explain.

6.6) A force is applied to an object initially at rest. The force acts over a distance d taking the object up to a speed v.

- **a.)** If the force had been halved but the distance remained the same, how would the final velocity have changed (if at all)?
- **b.)** If, instead, the distance had been halved with the force remaining unchanged, how would the final velocity have changed (if at all)?

6.7) What is the ONE AND ONLY thing potential energy functions do for you?

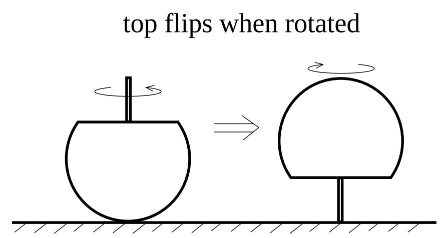
6.8) An ideal spring is compressed a distance x. How much more force would be required to compress it a distance $2x$? How much more energy would be required to execute this compression?

6.9) A mass moving with speed v strikes an ideal spring, compressing the spring a distance x before coming to rest. In terms of v, how fast would the mass have to be moving to compress the spring a distance $2x$?

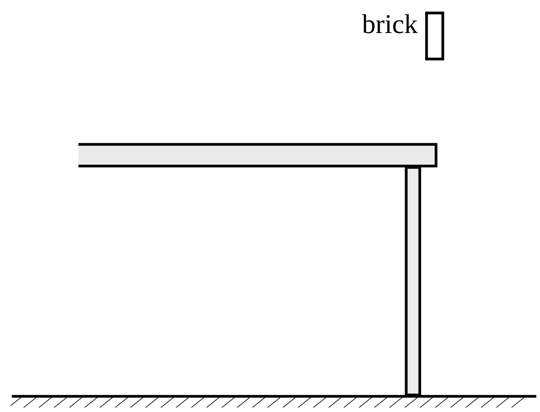
6.10) A simple pendulum (a mass attached to a swinging string) is pulled back to an angle θ and released. Ignore friction.

- **a.)** If the mass is doubled, what will happen to the velocity at the bottom of the arc?
- **b.)** If the length of the pendulum arm is doubled, how will the velocity at the bottom of the arc change?
- **c.)** Is there any acceleration at the bottom of the arc? If so, how much and in what direction?
- **d.)** How much work does tension do as the bob moves from the initial point to the bottom of the arc?
- **e.)** How much work does gravity do as the bob moves from the initial point to the bottom of the arc?

6.11) There is a toy on the market--a top--that, when spun, flips itself over (see sketch). What is the top really doing as it moves from the one state to the other state?



6.12) A brick is held above the edge of a table. Suzy Q looks at the brick, deduces that if it were to fall it would land ON the table, and calculates the brick's gravitational potential energy with that in mind. In doing so, she comes up with a number N_1. Big Jack, who happens to have terrible eyesight and has left his glasses at home, looks at the brick and decides that if it falls, it will land on the ground. He keeps that in mind as he calculates the brick's gravitational potential energy coming up with a number N_2. Which potential energy quantity is correct? Explain.



6.13) For a spring system, it is very obvious when there is no potential energy wrapped up in the position of the spring. For a gravitational situation near the surface of the earth, that isn't the case. What is the telltale *difference* between the two situations?

6.14) Is it possible for:

- **a.)** Potential energy to be negative? If yes, give an everyday example.
- **b.)** Kinetic energy to be negative? If yes, give an everyday example.
- **c.)** Work quantity to be negative? If yes, give an everyday example.
- **d.)** Power to be negative? If yes, give an everyday example.

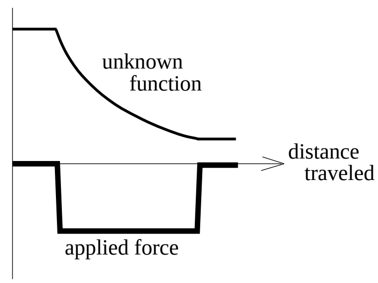
6.15) The units of power *could* be which of the following (more than one are possible)?

a.) Joules/sec.
b.) Watts/sec.
c.) $Kg \cdot m^2/s^3$.
d.) $Nt \cdot m/s$.

6.16) *Work* is to *energy* as *force* is to *velocity*. How so?

6.17) The potential energy function associated with a spring force of $-kx$ is $.5kx^2$. What would you expect the potential energy function for a force of $-kx^5$ to be? How would you derive such a function?

6.18) A vehicle moves in the *+x direction*. The net force applied to the vehicle is shown to the right along with a second graph. What might that second graph depict?

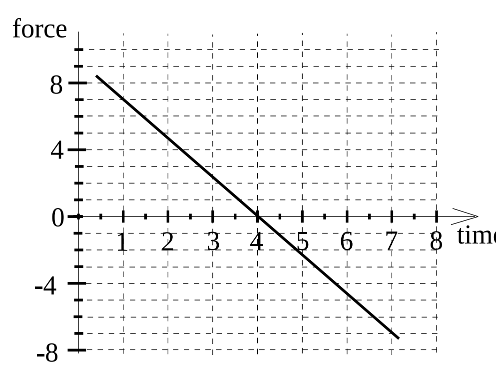


6.19) A force is applied to an object for some period of time *t*. During that time it does *W's* worth of work. If the time of contact remains the same but the force is doubled, what will the ratio of the work quantities be?

6.20) Assume you have a constant force $\mathbf{F} = (12 \text{ newtons})\mathbf{i}$ that does work on a moving object as the object travels a distance $\mathbf{d} = (2 \text{ meters})\mathbf{i}$ in time $t = 3$ *seconds*.

a.) At what rate is energy being pumped into the system?
b.) What is the name given to the quantity you derived in *Part a*?
c.) Come up with four different ways to express the quantity named in *Part b*.
d.) In the MKS system, what are the units for this quantity and what are the units called?

6.21) The graph shows the force $\mathbf{F}$ applied to an object that moves with a *constant velocity* of $.5\ m/s$ in the $-\mathbf{i}$ *direction*. Assuming $\mathbf{F}$ is oriented along the *x axis*:



a.) What can you say about the other forces that act in the system?
b.) How much power does $\mathbf{F}$ provide to the object between $t = 1$ *second* and $t = 7$ *seconds*?

c.) After $t = 4$ *seconds*, ***F's*** direction changes. What does that say about the power associated with ***F*** from then on?

d.) How much power, on average, does ***F*** provide between $t = 1$ *second* and $t = 4$ *seconds*?

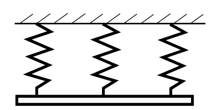
e.) As an interesting twist, given that the average power provided to the system between $t = 4$ *seconds* and $t = 7$ *seconds* is 1.75 watts, how much work does the force do during that period of time?

6.22) Let's assume that a car engine provides a constant amount of power. The car accelerates from zero to 30 m/s. Is the car's acceleration constant?

6.23) A group of students was asked the following question: "*In the real world*, what does the power requirement do as you double a car's velocity?" Assuming a reasonable answer was expected, what information *is missing* in the set-up? That is, what additional information would the students have needed to answer sensibly?

6.24) In his younger days, George boasted he could do a million joules of work. Gertrude, his betrothed, wasn't impressed. Why do you suppose she wasn't moved?

6.25) Three identical springs are attached at the ceiling. A bar of mass m is hooked to the group. If the new system's equilibrium position is d units below the springs' unstretched lengths, what must the spring constant be for each spring? Use *energy considerations* to dismantle this problem.



PROBLEMS

6.26) A 3 kilogram mass moving at 2 m/s is pulled 35 meters up a 25^o incline by a force F (see Figure I). If the coefficient of friction between the mass and the incline is .3:

a.) How much work does *gravity* do as the mass moves up the incline to the 35 meter mark?

b.) How much work does *friction* do as the mass moves up the incline to the 35 meter mark?

c.) How much work does the *normal force* do as the mass moves up the incline to the 35 meter mark?

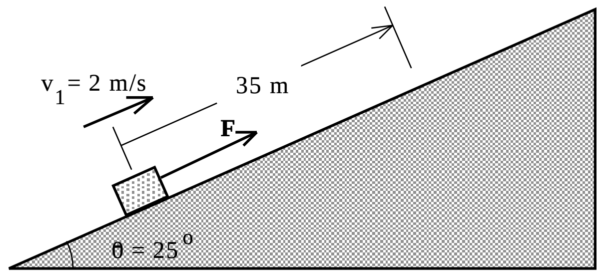


FIGURE I

d.) How much *kinetic energy* does the mass initially have?

e.) WORK/ENERGY PROBLEM: Assuming the mass's *velocity* at the 35 meter mark is 7 m/s, use the *work/energy theorem* to determine the force F. *Do this as you would on a test.* That is, forget for the moment that you have done any work above and lay this problem out completely in algebraic form before putting in numbers.

6.27) A force ***F*** is applied to a mass $m = .5\ kg$ as it proceeds up a frictional hemispherical dome of radius $R = .3$ *meters*. The force is ALWAYS at an angle of 12^o, relative to the mass's motion (see Figure II), and is always equal to $mg/4$ (that is 1/4 of the mass's weight). As the body moves from 20^o to 60^o up the dome, the distance traveled is .21 meters. How much work does ***F*** do on the mass as the body executes that motion? Put the numbers in last.

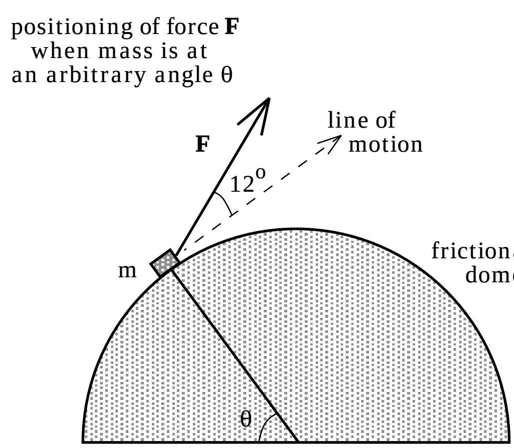


FIGURE II

6.28) How much energy is stored in a spring compressed 20 centimeters (.2 meters) if the spring's *spring constant* is $k=120\ nts/m$?

6.29) Tarzan ($m_T = 80\ kg$) stands on a 12 meter high knoll (see Figure III). He grabs a taut, 15 meter long vine attached to a branch located 17 meters above the ground and swings down from rest to Jane perched on a 5 meter high mole hill (they breed particularly big moles in Africa).

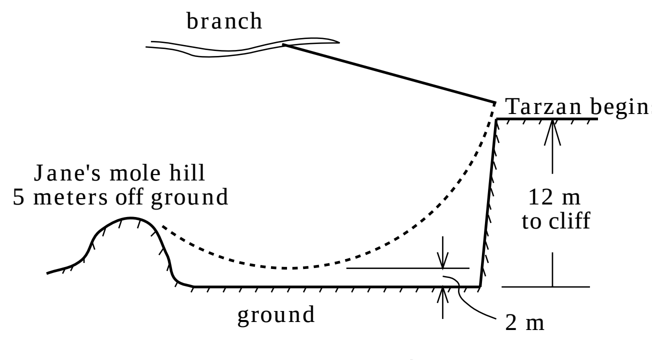


FIGURE III

a.) What is Tarzan's velocity when he reaches Jane?

b.) What is the tension in the vine when Tarzan is at the bottom of the arc? (Note: Tarzan is moving through a CIRCULAR path).

c.) What is the tension in the vine just before Tarzan lets go upon reaching Jane?

6.30) A 12 kilogram crate starts from rest at the top of a curved incline whose radius is 2 meters (see Figure IV). It slides down the incline, then proceeds 18 additional meters before coming to rest. What is the *frictional force* between the crate and the supporting floor (both curved and horizontal)? Assume this *frictional force* is constant throughout the entire motion.

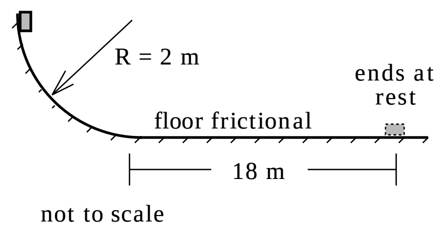


FIGURE IV

6.31) Pygmies use blow-guns and 15 gram (.015 kg) darts dipped in the poison curare to immobilize and kill monkeys that live in the tree-top canopy of their forest home. Assume a pygmy at ground level blows a dart at 85^o (relative to the horizontal) at a monkey that is 35 vertical meters up (over 100 feet). Assuming a dart must be moving at 4 m/s to effectively pierce monkey skin, what is the *minimum velocity* the dart must be moving as it leaves the blow-gun if it is to pierce the monkey?

6.32) A freewheeling 1800 kilogram roller coaster cart is found to be moving 38 m/s at *Point A* (see Figure V). The actual distance between:

Point A and *Point B* is 70 meters;
Point B and *Point C* is 60 meters;

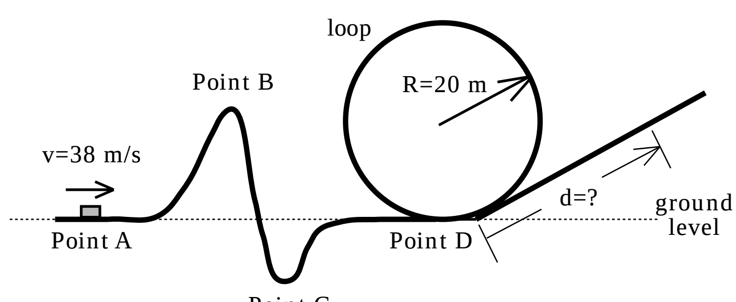


FIGURE V

Point C and *Point D* is 40 meters, where *D* is just before the cart enters the loop.

If the average frictional force acting throughout the motion is 27 newtons, the radius of the loop is 20 meters, the first hill's height 25 meters, the first dip 15 meters, and the incline just after the loop coming directly off the loop's bottom at an angle of 30^o:

a.) How fast is the cart moving at *Point C*?

b.) How far up the incline *d* will the cart travel before coming to rest?

c.) What must the cart's *minimum velocity* be at *Point A* if it is to just make it through the top of the loop without falling out of its CIRCULAR MOTION (hint, hint).

Note: The phrase "just making it through the top" means that for all intents and purposes, the *normal force* applied to the cart by the track goes to zero leaving gravity the only force available to affect the cart's motion at the top.

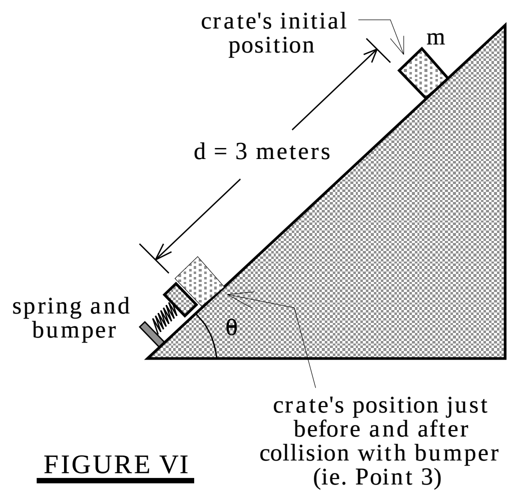


FIGURE VI

6.33) A spring-loaded bumper is placed on a 55^o frictional incline plane. A 60 kilogram crate breaks loose a distance 3 meters up the incline above the bumper and accelerates down the incline (see Figure VI). If the average *frictional force* applied to the crate by the incline is 100 newtons and the spring constant is 20,000 newtons/meter:

a.) How much will the bumper spring compress in bringing the crate to rest? (Assume there is friction even after the crate comes in contact with the bumper).

b.) The crate compresses the bumper's spring which then pushes the crate back up the incline (the crate effectively bounces off the bumper). If a total of three-quarters of the crate's *kinetic energy* is lost during the collision, how far back up the incline will the crate go before coming to rest?

6.34) Because *gravitational attraction* between you and the earth becomes less and less as you get higher and higher above the earth's surface, the *gravitational potential energy function* for a body of mass m_1 that is a substantial distance *d units* away from the earth's surface is not *mgh*; it is:

$$U = - Gm_1m_e/(r_e+d),$$

where *G* is called *the universal gravitational constant* (6.67×10^{-11} $m^3/kg \cdot s^2$), m_e is the mass of the earth (5.98×10^{24} kg), and r_e is the radius of the earth (6.37×10^6 m).

A satellite is observed moving at 1500 m/s when 120,000 meters above the earth's surface. It moves in an elliptical path which means its height and velocity are not constants. After a time, the satellite is observed at 90,000 meters. Ignoring frictional effects, how fast is the satellite traveling at this second point?

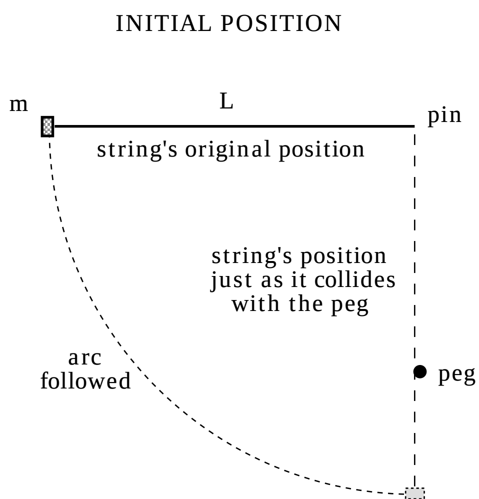


FIGURE VIIa

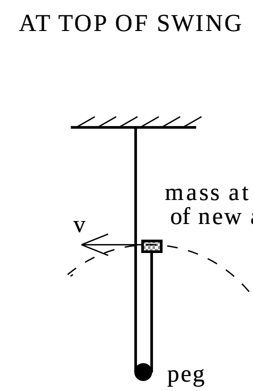


FIGURE VIIb

6.35) A string of length *L* is pinned to the ceiling at one end and has a mass *m* attached to its other end. If the mass is held in the horizontal and released from rest, it freefalls down through an arc of radius *L* until the string collides with a peg located a distance *L/3* from the bottom of the arc (see Figure VIIa). From there it proceeds along an arc of lesser radius (i.e., a radius of *L/3*). Assuming one-tenth of the energy in the system is lost during this collision, what will the tension *T* in the string be as the body moves through the *top* of its final arc (see Figure VIIb)?

6.36) Revisiting *Problem 2*, let's assume the average normal force is *.4mg* over the run between the angles 20^o and 60^o up the dome. How fast is the mass going when it gets to the 60^o mark if it starts from rest? You can

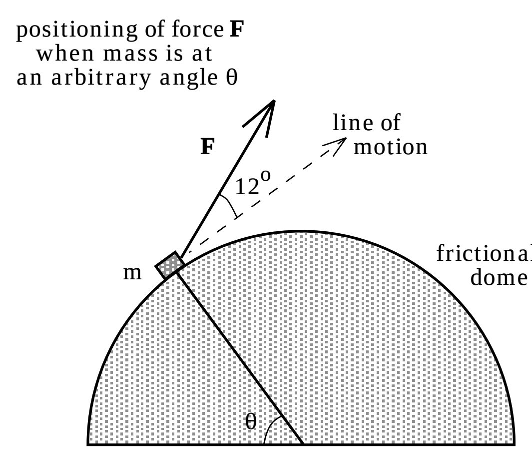


FIGURE II

assume a coefficient of friction between the mass the and dome of *.6*.